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Discussion

# Discussion of "A simple method to determine breaker height and depth for different deepwater wave height/length ratios and sea floor slopes", by J.P. Le Roux [Coastal Engineering 54 (2007) 271–277]

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**Abstract**

In the recent paper by J.P. Le Roux [Coastal Engineering 54 (2007) 271–277], the author provides a simplified approach to calculating the depth, length, and height of waves at the onset of depth-induced breaking (i.e. at the breaker line). However, the proposed methodology and the comparisons to other methods suffer from a large number of inconsistencies and basic calculation errors. In addition, there are a number of erroneous physical interpretations and many of the conclusions are based on erroneous data. The remaining conclusions are either not new or based on circular logic, such as to render them moot. In the following, we will not attempt to point out all the errors or inconsistencies that we found, instead we focus on major points of contention.


*Keywords:* Waves; Wave breaking; Shoaling

## 1. Inconsistent methodology

The methodology presented in this paper essentially consists of the following steps:

(1) For fully developed wind waves the offshore wave steepness is equal to a constant value such that

$$H_0/L_0 = (9\pi)^{-1} \qquad \text{(LR, Eq.18)}$$

where $H_0$ and $L_0$ are the deepwater wave height and wavelength, respectively. Note, hereafter equations given in the original work by Le Roux will be identified as (LR, Eq. xx).

(2) The shoaling of nonlinear waves is calculated using the model given by Sakai and Battjes (1980) (LR, Eqs. 20–24).

(3) The location of the onset of wave breaking is given by the following equation (based on data from the Shore Protection Manual, 1984):

$$H_b = d_b\left(-0.0036\alpha^2 + 0.0843\alpha + 0.835\right) \qquad \text{(LR, Eq.25)}$$

where $H_b$ is the wave height at breaking, $d_b$ is the breaking depth, and $\alpha$ is the bottom slope in degrees.

The main motivations given by the author for this work are given as:

"Most [previous] equations express the breaker height/breaker depth ratio ($H_b/d_b$) as a function of other variables, which means that either the breaker height is required to obtain the depth, or vice versa. A second shortcoming of existing methods is that they do not employ all the variables affecting the breaker height and depth, with the result that they apply only to limited conditions."

While the first statement is true in a literal sense, in practice it is recognized that equations for $H_b/d_b$must be used in conjunction with shoaling/refraction/diffraction models that predict local wave heights and local wave height to water depth ratios throughout a domain of interest. Hence, coupled iteratively with a

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wave propagation model, such equations for breaker height or depth can be used to predict both $H_b$ and $d_b$ individually. This is no different from the methodology presented by the author, which couples the shoaling model (LR, Eqs. 20–24) of Sakai and Battjes (1980) with Eq. (LR, 25).

In regards to the second statement, it is to the author's credit that the chosen shoaling model incorporates wave nonlinearity through a dependence on deepwater wave steepness, $H_0/L_0$. Hence, wave steepness is considered in the shoaling model and bottom slope is included in the breaking criterion. Nonetheless, the majority of conclusions drawn in the paper (e.g. LR, Eqs. 19, 26–30, and a number of the stated conclusions) apply only to fully developed wave conditions and nearly horizontal bottom slopes, thus likewise only applying to limited conditions.

We grant that the effort to utilize a shoaling model that incorporates nonlinearity is worthwhile; yet, the overall methodology and methods of comparison still exhibit significant inconsistencies. For example, Eq. (LR, 25) is an empirical equation calibrated to monochromatic laboratory data, which is inconsistent with the application of predicting the nearshore breaking condition for fully developed (or developing) wind seas. Further, in contrast to the range $0.83 < H_b/d_b < 1.33$ that is predicted by Eq. (LR, 25), irregular wave observations from field beaches have shown that $H_b/d_b$ (here we also denote this as $\gamma_b$ for convenience), can fall in the range $0.3 < \gamma_b < 1.1$ (e.g. Thornton and Guza, 1982; Sallenger and Holman, 1985; Raubenheimer et al., 1996). A typical value would be 0.42 when $H_b$ is taken as the root-mean-square wave height of an irregular sea (Thornton and Guza, 1983).

Even if we allow for the moment that the assumption $H_0/L_0 = (9\pi)^{-1}$ is reasonable and also allow that Eq. (LR, 25) is representative of breaking conditions under field conditions, the method of comparison of the proposed methodology to previous results as embodied in Table 2 in Le Roux (2007) is inherently inconsistent. Table 2 (LR) compares predictions of breaking wave heights and water depths from his methodology to the results of a number of pre-existing methods. Since the pre-existing methods (LR, Eqs. 1–16) consist only of breaking criteria (not shoaling models), a proper comparison of the various breaking criteria would require first using the shoaling equations (LR, Eqs. 20–24) to predict the wave heights ($H_w$) throughout the shoaling zone. Then, based on the shoaled wave heights at each depth, the breaking locations and wave conditions would be found *independently* from Eqs. (LR, 1–16) and Eq. (LR, 25).

Instead, the author has inconsistently mixed together the various breaking criteria. For example, using the shoaled wave heights and Eq. (LR, 25) the author has determined the breaking conditions for the proposed methodology (i.e. $H_b$, $d_b$, and thus $\gamma_b$; see columns labeled "LR" in his Table 2). Next, the author plugged in these predicted $d_b$ values into Eq. (LR, 2) to purportedly calculate the breaking wave heights predicted by Collins (1970) (see LR, column "Col — $H_b$"). The same technique was also used for Eqs. (LR, 12–15) (see LR, column "S&B2 — $H_b$"). Conversely, the author plugged in the predicted $H_b$ values from the proposed methodology into Eqs. (LR, 4–6) from (Weggel, 1972) in order to calculate the breaking depths (LR, column "Weg — $d_b$"). Not only are these comparisons totally inconsistent, they are not even meaningful. Based on a given bottom slope (e.g $\alpha = 1 \times 10^{-6\circ}$) alone, Eq. (LR, 25) gives $\gamma_b = 0.835$ while Eq. (LR, 2) gives $\gamma_b = 0.72$. It is inherently obvious that if you multiply the $d_b$ predicted from Eq. (LR, 25) by the quantity 0.72 one ends up with a smaller wave height than that given by Eq. (LR, 25). Hence, the author arrives at the conclusion that "the equation of Collins (1970) …appears to underestimate breaker heights somewhat over a nearly horizontal bottom". Examination of Eqs. (LR, 2) and (LR, 25) (without using the shoaling model) for a bottom slope of 5° shows that the relationship between $\gamma_b$ is reversed between the two (i.e. 1.21 to 1.17, respectively); hence, the calculated $H_b$ from Eq. (LR, 2) are now slightly higher than those from Eq. (LR, 25) and are deemed to "correlate well the method proposed here on a slope of 5°". It can be seen straightforwardly by comparing Eqs. (LR, 2) and (LR, 25) analytically that they lie within 5% of each other in the region $3° \leq \alpha \leq 5°$.

Table 1
Corrected version of Table 2 Le Roux (2007)

| $T_w$ | Col | S&B1 | S&B2 | K&G | Kom | LR | Weg | Kom | LR |
|---|---|---|---|---|---|---|---|---|---|
| | $H_b$ | $H_b$ | $H_b$ | $H_b$ | $H_b$ | $H_b$ | $d_b$ | $d_b$ | $d_b$ |
| $1 \times 10^{-6\circ}$ slope | | | | | | | | | |
| 1.6 | **0.16** | 0.17 | **0.20** | 0.15 | 0.16 | 0.17 | **0.21** | 0.00 | 0.20 |
| 3.3 | **0.67** | 0.71 | **0.60** | 0.66 | 0.66 | 0.71 | **0.89** | 0.01 | 0.85 |
| 5.1 | **1.61** | 1.69 | **1.44** | 1.57 | 1.58 | 1.69 | **2.12** | 0.02 | 2.03 |
| 6.6 | **2.70** | 2.82 | **2.41** | 2.63 | 2.64 | 2.83 | **3.55** | 0.04 | 3.39 |
| 8.4 | **4.37** | 4.57 | **3.90** | 4.26 | 4.28 | 4.59 | **5.75** | 0.06 | 5.50 |
| 11.1 | **7.63** | 7.99 | **6.80** | 7.43 | 7.48 | 8.02 | **10.03** | 0.11 | 9.60 |
| 11.8 | **8.62** | 9.03 | **7.69** | 8.40 | 8.45 | 9.06 | **11.34** | 0.13 | 10.85 |
| 5° slope | | | | | | | | | |
| 1.6 | **0.19** | | | | | 0.19 | | | 0.16 |
| 3.3 | **0.82** | | | | | 0.81 | | | 0.69 |
| 5.1 | **1.96** | | | | | 1.93 | | | 1.65 |
| 6.6 | **3.28** | | | | | 3.23 | | | 2.77 |
| 8.4 | **4.37** | | | | | 5.23 | | | 4.49 |
| 11.1 | **9.28** | | | | | 9.14 | | | 7.83 |
| 11.8 | **10.49** | | | | | 10.33 | | | 8.85 |
| 10° slope | | | | | | | | | |
| 1.6 | | | | | | 0.20 | | | 0.15 |
| 3.3 | | | | | | 0.85 | | | 0.65 |
| 5.1 | | | | | | 2.04 | | | 1.55 |
| 6.6 | | | | | | 3.41 | | | 2.59 |
| 8.4 | | | | | | 5.53 | | | 4.19 |
| 11.1 | | | | | | 9.65 | | | 7.32 |
| 11.8 | | | | | | 10.91 | | | 8.27 |

Corrected values shown in bold.

We emphasize here that we are not calling into question the relative differences in the breaking conditions that would be predicted by the proper use of Eq. (LR, 2) and Eq. (LR, 25). Clearly, the smaller $\gamma_b$ of Eq. (LR, 2) will predict waves to break further offshore and at a lower wave height than with $\gamma_b = 0.835$. However, the actual breaking wave heights or water depths given for many of the models listed in (LR) Table 2 are not correct. A corrected (partial) version of (LR) Table 2 is given here in our Table 1. Note the corrected version also negates the author's conclusion that "Collins (1970) consistently yields the lowest breaking height."

## 2. Calculation errors

- In (LR) Table 1 the author lists "Observed wave periods and deepwater fully developed wave heights…derived from nomograms (Figs. II-2-25 and II-2-26) in Resio et al. (2003)." These data are then used to establish the relationship $H_0/L_0 = (9\pi)^{-1}$ for fully developed seas.

The nomograms given in Resio et al. (2003) are for duration-limited conditions. The values listed in (LR) Table 1 appear to correspond approximately to the limiting values of those nomograms at large durations. However, quoting from Resio et al. (2003), "the curves in these nomograms are based on Eqs. II-2-30 and II-2-36 through II-2-38". Eq. II-2-37 in Resio et al. gives the fully developed conditions and reads:

$$\frac{gH_0}{u_*^2} = 2.115 \times 10^2 \quad \text{and} \quad \frac{gT}{u_*} = 2.398 \times 10^2$$

where $u_*$ is the friction velocity, and some of the notation has been modified to be consistent with the present discussion. Using these two equations, and $L_0 = gT^2/2\pi$, it can be shown that the fully developed condition should be approximately $H_0/L_0 = (14\ \pi)^{-1}$.

- Much of the data in (LR) Table 3 is incorrect.

There are a number of errors in (LR) Table 3, some are errors of consistency some are just plain calculation errors. (LR) Table 3 is intended to show how breaking conditions change when the incident wave condition changes from a fully developed to a developing sea. To account for a developing sea the author states that "...the same deepwater wave heights were used as in Table 2, but the $H_0/L_0$ ratios were increased to 0.05 by shortening the wavelength". Yet, the wave periods listed in (LR) Table 3 are the same as in (LR) Table 2, *even though the deepwater wavelengths have changed*. Hence, the wave periods listed in (LR) Table 3 are not correct, which also means the $H_b$ values calculated from Eq. (LR, 7) (column "Kom – $H_b$") are incorrect. In addition, the calculated $H_b$ for Eqs. (LR, 9–11) and Eq. (LR, 3) are the same as in (LR) Table 2, which is obviously incorrect as they depend explicitly on deepwater steepness. We were unable to reproduce the results given for Eqs. (LR, 12–15) in (LR) Table 3, though we tried 1) accounting for the incorrect wave periods, 2) using the $d_b$ from Eq. (LR, 25), and 3) using the shoaling wave data correctly. There may be errors in other columns, as we have not checked them all. There are so many errors in (LR) Tables 2 & 3 the reader would be advised to recalculate everything independently.

## 3. Errors in physical reasoning

- On page 273 the author states that $L_b$ should be shorter on steep slopes because "the...wave crest seaward [of the breaker line] would still be in deep water. This crest would therefore be affected less by bottom friction, *advancing faster than...on a nearly horizontal slope. $L_b$ should therefore be shorter on steep slopes than on gentle slopes"* (emphasis ours).

First, it is irrelevant what is happening to the crest that is one wavelength offshore from the breaker line, what is of interest is the wavelength ($L_b$) when the crest reaches the breaker line. Secondly, bottom friction has nothing to do with the physical process of shoaling and the decrease in wavelength (except in some special friction-dominated cases). Thirdly, if a wave were "advancing faster" due to any given process, then it would necessarily have a *longer* wavelength because wave celerity equals the ratio of wavelength to wave period.

- On page 274 the author states "...wave breaking...would not occur on an absolutely horizontal bottom".

This is patently untrue. In addition, a number of the breaking criteria used in this paper predict breaking conditions on a horizontal slope.

- On page 276 the author states that shoaling waves from developing seas (i.e. waves with increased wave steepness) will "[manifest] as plunging instead of spilling breakers on the same slope."

This statement is incorrect based on existing understanding. Breaker type is known to be partially related to offshore wave steepness (and bottom slope) through the surf similarity parameter, $\xi_0$, given by:

$$\xi_0 = \frac{\tan\alpha}{\sqrt{H_0/L_0}}$$

The lowest values of $\xi_0$ are associated with spilling breakers, and values of $0.5 < \xi_0 < 3.3$ are associated with plunging breakers (see Eqs. II-4-1 and II-4-2 in Smith, 2003). Hence holding bottom slope constant, the increased offshore steepness of developing waves should tend to decrease $\xi_0$ and tend to lead to spilling rather than plunging.

## 4. Circular arguments/trivial conclusions

- Numerous times results are stated to be "in accordance with laboratory observations (Shore Protection Manual, 1984)". For example on p. 274 (Le Roux, 2007) it is stated that:

"Finally it can be shown from Eqs. (26) and (19) that

$$H_b/d_b = 1/1.2 = 0.8333 \tag{30}$$

Eq. (30) agrees with the experimental value of 0.83 reported for a 0° slope in the[SPM] ...".(The argument that the results from the proposed methodology agree with the SPM is repeated three separate times on page 275.).

First, the author must intend to refer to Eq. (LR, 27) instead of Eq. (LR, 19). Most egregiously, this is a circular argument. Of course the predicted $H_b/d_b$ agrees with the SPM value. The values $H_b$ and $d_b$ were arrived at through the use of Eq. (LR, 25), *which is a curve fit to the SPM data*. As stated previously, for $\alpha = 1 \times 10^{-6\circ}$ Eq. (LR, 25) gives $\gamma_b = 0.835$. The difference between 0.83, 0.8333, and 0.835 can be attributed to the precision used in the author's spreadsheet.

- The proposed method along with Collins (1970) and Fenton and McKee (1990) predict a decrease in breaking heights for developing waves compared to fully developed waves.

In fact, all breaking models should predict this, and where (LR) Table 3 does not show lower breaking heights than (LR) Table 2 it is only because of the aforementioned calculation errors. The fact that lower breaking heights are predicted for developing waves can be explained by the fact that here what is intended by "developing waves" is that the offshore wavelength (and, hence, wave period) were relatively decreased, whilst offshore wave heights were held constant. Thus, regardless of the breaking criterion used, the decrease in the wave period will cause the break point to move closer to shore (even for breaking conditions that vary with $H_0/L_0$, when used properly) as long as any reasonable shoaling model is used.

- For the proposed method, $H_b/d_b$ is the same for high-steepness waves and for fully developed waves (0.8315 and 0.8333, respectively).

Besides the fact that this result contradicts previous observations that show that $\gamma_b$ has a dependence on deepwater steepness, in this case this is a trivial result as it arises directly from the use of the Eq. (LR, 25), which is independent of deepwater steepness. In fact, the calculated values of $\gamma_b$ from the proposed method should be exactly the same for both conditions, the slight differences between the given values are only due to slight differences in the (unnecessary) iterative method used for their calculation.

- $H_b/d_b$ increases with slope angle.

This is a trivial result that again arises directly from the use of Eq. (LR, 25), which explicitly states that $H_b/d_b$ increases with slope angle.

## 5. Other erroneous conclusions

- "For a nearly horizontal bottom, Eqs. (LR, 20 and 25) yield a $H_b/d_b$ ratio of 0.83, which is considerably higher than the ratios of 0.71–0.78 normally considered to be the breaking limit…"

It is not necessary to use Eq. (LR, 20) to see that Eq. (LR, 25) yields a $H_b/d_b$ ratio of 0.835 for a horizontal bottom. Also, there is hardly a consensus on 0.71–0.78 being the "breaking limit", numerous observations show that $H_b/d_b$ can exceed 1.0, especially for plunging breakers.

- The author states that the difference between the $H_b/d_b$ ratios from the different breaking models can be explained using cnoidal wave theory and by modifying the breaking depth in an ad-hoc manner.

The basis of the author's reasoning is that, if we consider an example cnoidal wave in a breaking depth (i.e. still water level) of $d_b$=3.20 m, with water depth under the crest of 5.52 m and water depth under the trough of 2.59 m, then the proper $H_b/d_b$ ratio is obtained by artificially adjusting the still water level to the mid-point between crest and trough, i.e. changing $d_b$ from its original value of 3.20 m to 4.06 m. This idea is completely preposterous, as the mid-point between crest and trough has no physical meaning in an asymmetric wave. The concept that in the breaker zone the local ratio of wave height to water depth should take on a constant value is based on a vast number of observations that consisted of measurements of wave heights (crest to trough distances) as well as water depths. The author is implying that the water depths *measured and used* in developing Eqs. (LR, 1–16 and 25) need to be adjusted to account for the asymmetry of the crest and trough levels about the still water line. In that case *all* of the breaker formulas would have to be recalibrated (and then we would be back at square one and have other differences between predicted and measured $H_b/d_b$ to try to explain).

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