

# Centralized Carrier Collaboration Multihub Location Problem for Less-Than-Truckload Industry Hybrid Hub-and-Spoke Network

Salvador Hernández, Avinash Unnikrishnan, and Satyen S. Awale

**A centralized carrier collaboration multihub location problem (CCCMLP) for the small to medium-sized less-than-truckload industry is addressed. In the CCCMLP, a central entity (e.g., a third-party logistics firm) seeks a set of collaborative consolidation transshipment hubs to establish a hybrid collaborative hub-and-spoke system that minimizes the total collaborative costs for the set of collaborating carriers. Previous studies focused on addressing the exchange of capacity without considering the location of transfer hubs and the routes that connect them. A carrier has the option either to collaborate or to ship its demand directly without collaborating. The decision depends on the expected profit margin over shipping directly while following a revenue-generating, rate-setting behavior. The CCCMLP was formulated as a variant of the P-hub location problem, which is NP-hard and solved with Lagrangian relaxation. Numerical experiments were conducted to gain insight into the performance of the CCCMLP formulation under various network sizes and numbers of hubs. The results indicate that larger expected profit margins from collaborative carriers applying revenue-generating behavior would increase the likelihood of collaboration by carriers. As the network size increases, the effect of hybrid hub location costs drops.**

Since the advent of the Internet in the 1990s, the less-than-truckload (LTL) industry has become increasingly competitive. Shippers, usually larger manufacturers and retailers, have increased their transportation requirements because of innovative inventory practices and increased activity in e-commerce, which has spurred competition (1). In addition, the Internet, along with information communication technologies (ICT), is prompting changes to the structure of transportation marketplaces by fostering more spatially spread demand (2). These innovations have created new challenges for LTL carriers in the form of increased costs related to deadheading (moving empty trucks) and increased energy prices. The greatest economic impact has been felt in small to medium-sized LTL trucking, which has endured increased costs that affect the ability to sustain profits. Low margins

of profitability, spatially spread demand, and intense competition have incited a trend to seek solutions through ICT and the Internet (3). One manifestation of this is an increase in collaboration among small to medium-sized LTL carriers. That is, small to medium-sized carriers have begun to develop strategies that exploit synergies (such as excess capacity, overlapping lanes, and facilities), which form the basis for some forms of collaboration.

LTL carrier collaboration can improve operations and reduce supply chain costs. By collaborating, smaller and medium-sized LTL carriers can increase use of assets (such as capacity or facility space) and strengthen their market position. The challenge for a collaborative effort is to balance multiple requests by LTL carriers that require resources with the available transportation capacity to service those requests. This balance depends on the affordability of transportation services provided to collaborative member carriers, as well as on the shipment size and value (4–6). Carrier collaboration is highly dependent on location factors, that is, where to transfer loads to fulfill demand requirements. This is important because smaller and medium-sized LTL carriers operate within a point-to-point network structure of warehouses, depots, and distribution centers. Point-to-point networks move LTL shipments directly between facilities, such as end-of-line terminals, without intermediate stops to consolidate loads. Therefore, this paper investigates the benefits of locating transfer hubs to facilitate carrier collaboration. Here, “hybrid” refers to the short-term transformation of an LTL point-to-point network into a type of hub-and-spoke with direct routes.

A major challenge in carrier collaboration is to identify potential locations for a consolidation hub to facilitate transfers of loads in the collaborative. Identification of these hubs depend on such factors as contractual agreements for using the facility and costs of holding, collaboration, and congestion. Contractual agreement costs may pertain to the handling of the transfer. These costs can be either fixed or variable, and fixed costs can be per unit, per weight, or per volume. In addition, these costs may depend on the transfer point (for example, city) in which they occur, as well as incoming and outgoing trucks, for example, the cost of the crew unloading or loading the trailer and any cost associated with the operation of the vehicle (7). Collaborative holding and congestion costs are available elsewhere (5, 6). In addition, selection of hub locations may depend on product handling and storage capabilities, for example, refrigerated storage or humidity-controlled areas for specific goods. Consideration of such heterogeneous products increases the complexity of the problem (4, 5). This study assumes a homogeneous fleet that handles a single product type (e.g., nonperishable goods).

S. Hernández and S. S. Awale, Department of Civil Engineering, University of Texas at El Paso, 500 West University Avenue, El Paso, TX 79968. A. Unnikrishnan, Department of Civil and Environmental Engineering, West Virginia University, Room 621 ESB, P.O. Box 6103, Morgantown, WV 26506-6103. Corresponding author: S. Hernández, shernandez43@utep.edu.

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This study addresses and establishes a framework for a centralized carrier collaboration multihub location problem (CCCMLP) among a group of small to medium-sized LTL carriers. Here, a central entity (such as a third-party logistics firm) facilitates collaboration among member carriers to minimize total (collaborative) system costs subject to a rate-setting behavior of the individual carriers. The carriers seek a set of collaborative consolidation transshipment hubs to service various shipments. The CCCMLP is addressed from a planning perspective, in that the demand, facility capacities, and carrier collaborative rates are known a priori. The costs associated with congestion caused by both traffic and terminal delays are captured through holding costs that vary with the location of hub transfer facilities. The rate-setting behaviors exhibited by the carriers follow those presented by Hernandez and Peeta (4).

## LITERATURE REVIEW

### Carrier Collaboration

LTL carrier collaboration is a little-explored concept within the ground freight domain. Previous studies focused on collaboration within the truckload carrier, liner shipping, and rail industries (1, 8–12). Hernandez and others introduced and examined the viability of LTL carrier collaboration within static and dynamic contexts for a single carrier and centralized planning perspective for multiple carriers (4–6). Their studies explored the potential benefits of LTL carrier collaboration based on the degrees of collaboration and for the centralized work rate-setting strategies. In addition, Voruganti et al. studied partial and full collaboration among carriers and used the Shapley value principle to distribute the profits (12). The network topology was found to have a significant impact on the success of the collaboration. Bailey et al. developed integer programming models and heuristic algorithms that can be used by medium-level freight companies to evaluate savings obtained by entering into collaboration with other shippers and carriers to minimize deadheading (13). Bailey also developed a cardinality and capacity constrained lane-covering formulation for shipper collaboration and used a tabu search to solve the problem (14). Proportional and marginal cost-based allocation mechanisms were used to distribute transportation costs among participating shippers. However, these studies did not focus on network design. In addition, the location of hubs to facilitate collaboration was not addressed as a separate objective component.

### Hybrid Hub-and-Spoke and Hub Location

The concept of hybrid hub-and-spoke is relatively new. Although not explicitly collaboration, hybrid hub-and-spoke for a single LTL carrier trying to minimize transportation costs was introduced by Zhang et al. (15). In that work, “hybrid” refers to the addition of direct routes to a pure hub-and-spoke system. The authors formulated the problem as a combinatorial one and solved it by using a genetic algorithm. Similarly, Zäpfel and Wasner developed a hub-and-spoke system for cooperative third-party logistics firms (16).

From an LTL and pure hub-and-spoke perspective, Cunha and Silva focused on configuring a hub-and-spoke network for an LTL trucking company in Brazil (17). They sought to determine the number of consolidation terminals (hubs), their locations, and the assignment

of the spokes to the hubs while minimizing the total cost. The authors used a genetic algorithm and local improvement procedure to solve the problem.

From a pure hub-and-spoke perspective, O’Kelly formulated and solved a hub-and-spoke facility location problem as a quadratic integer program and developed enumeration heuristics to solve the problem (18, 19). The initial formulation was developed along multiple directions so that new features could be added consistent with various applications of hub-and-spoke facilities. The studies of Aykin (20), Ernst and Krishnamoorthy (21), and Ebery et al. (22) considered capacities on hubs, whereas Pirkul and Schilling (23), Klineciewicz (24), and Topcuoglu et al. (25) studied the uncapacitated case.

Problems of hub-and-spoke facility locations can be classified as single assignment (21), where a specific origin–destination flow is assigned to one hub only, and multiple assignment (22), where a specific origin–destination flow is split among multiple hubs. The studies by Kuby and Gray (26) and Aykin (27) developed models for hybrid hub-and-spoke facility location that captured the flexibility of flows being sent directly without passing through hubs. Elhedhli (28) and Elhedhli and Wu (29) developed a nonlinear programming formulation to capture congestion at hubs.

Lagrangian relaxation has been widely used to solve variations of the hub-and-spoke facility location problem (20, 23, 28, 29). Other solution techniques that have been explored are heuristic methods (24, 30), metaheuristics (25, 31), and exact methods such as branch and bound (20, 24, 27).

In the context of the CCCMLP, the current literature either addresses collaboration without consideration of multihub location or addresses the multihub location in the context of a single LTL carrier. The present paper defines the hybrid hub-and-spoke system as a set of collaborative consolidation transshipment hubs from a current point-to-point network structure; a hub-and-spoke system is formed without costly investment in new facilities. A centralized collaborative network of carriers can benefit from the hub-and-spoke system by consolidating shipments at specified locations to increase the efficiency of operations for the member carriers. This is the first study to model a centralized carrier collaboration problem for the development of a hybrid hub-and-spoke system. In addition, this work differentiates itself from previous hub location literature by assuming a real-world rate-setting behavior strategy. In planning, the CCCMLP is a starting point for studying the effects of rate setting by collaborative carriers in a centralized carrier multihub collaborative network.

## MATHEMATICAL MODEL OF CCCMLP

### Problem Description and Assumptions

The CCCMLP is used to determine a set of hybrid collaborative consolidation transshipment hubs for a central entity (e.g., third-party logistics firm) to help establish a collaborative hybrid hub-and-spoke system that minimizes the total collaborative costs for the set of collaborating carriers. Hence, a carrier in this system is classified as either collaborative (shares the costs to set up hybrid hubs) or noncollaborative (chooses to ship directly). The operational networks of the collaborating carriers can be identical geographically or can overlap in some segments relative to other carriers in the collaborative.

The collaborative rate structure of the collaborative carriers is represented by revenue-oriented behavior. If a collaborative opportunity cannot be identified for hybrid collaborative consolidation transshipment hubs, a noncollaborative option is considered. The costs of shipping is assumed to fall on the carrier itself.

The following assumptions are made in the CCCMLP: (a) candidate hybrid collaborative consolidation transshipment hubs are uncapacitated and (b) homogeneous products are shipped. In addition, the problem is deterministic in that the demand is known and the available holding times at facilities are time invariant. By contrast, a stochastic version of the problem would entail stochasticity of demand of the collaborating carriers.

## Problem Formulation

### Sets

Let a shipment from collaborative carrier  $q \in Q$  enter the collaborative network through an origin facility  $i \in I \subseteq N$  and travel via hybrid collaborative consolidation candidate transshipment hubs  $l, m \in N$  and exit through a destination facility  $j \in J \subseteq N$ . For each collaborative carrier  $q \in Q$  shipment, its origin facility  $i \in I$  and its destination facility  $j \in J$  constitute its origin–destination pair. Here, an origin and destination facility can represent a supplier or warehousing or a retailer.

### Parameters

Each collaborative carrier  $q \in Q$  has an associated demand  $d_{ijq}$ . Let  $\varsigma_{ijlm}$  be the collaborative carrier  $q \in Q$  revenue-oriented cost associated to a unit of demand  $d_{ijq}$  to travel between origin facility  $i \in I$  and destination facility  $j \in J$  when going via hybrid collaborative consolidation candidate transshipment facilities at node  $l \in N$  and  $m \in N$ . The revenue-oriented cost structure follows the work of Hernandez and Peeta (4) and is represented here as the functional form

$$\varsigma_{ijlm} = \varsigma_{il} + \delta \varsigma_{lm} + \varsigma_{mj} \quad (1)$$

In Equation 1,  $\delta$  represents the collaborative discount (falls between 0 and 1) between hybrid consolidation collaborative candidate transshipment facilities  $l, m \in N$ . The discount rate as described by Hernandez and Peeta is composed of transfer rates per shipment and line haul costs (4).

The number of hybrid facilities the centralized carrier collaborative network wishes to locate is  $p$ . The cost to a carrier to establish a hybrid collaborative consolidation candidate transshipment hub is  $\phi_{iq}$  and is as follows:

$$\phi_{iq} = \vartheta_{iq} + \phi_i \quad (2)$$

In Equation 2,  $\vartheta_{iq}$  represents the holding costs as defined by Hernandez et al. (captures congestion effects) (5), and  $\phi_i$  represents the costs associated with establishing connections at a hub.

A comparative measure to the collaboration, the noncollaborative costs of moving a shipment of demand  $d_{ijq}$  between an origin facility  $i \in I$  to a destination facility  $j \in J$  directly is  $w_{ijq}$ . These costs comprise distance, labor (includes deadheading), and fuel costs associated with the shipment.

### Variables

If a shipment originating from  $i \in I$  headed to destination  $j \in J$  by collaborative carrier  $q \in Q$  travels via consolidation hubs at node  $l \in N$  and  $m \in N$ ,  $Y_{ijlmq}$  takes value 1 and takes 0 otherwise. This variable represents collaborative carrier participation; participation is not mandatory.

If a shipment originating from  $i \in I$  heads to destination  $j \in J$  by carrier  $q \in Q$ , who does not participate in the collaboration,  $V_{ijq}$  takes the value 1 and takes 0 otherwise.

If a hybrid collaborative consolidation candidate transshipment hub is located at node  $l \in N$ ,  $X_l$  takes the value 1 and takes 0 otherwise.

### Constraints

The constraint set of the CCCMLP consists of two sets. The first set, Constraints 3, 4, 5, and 6, models the location of the hybrid collaborative consolidation candidate hub. The second set, Constraints 4 and 7, establishes lower bounds on the revenue potential for the carrier collaborative network. The constraints are as follows:

$$\sum_{l \in N} X_l = p \quad (3)$$

$$\sum_{l \in N} \sum_{m \in N} Y_{ijlmq} + V_{ijq} = 1 \quad \forall i \in I, j \in J, q \in Q \quad (4)$$

$$\sum_{m \in N} Y_{ijlmq} \leq X_l \quad \forall i \in I, j \in J, l \in N, q \in Q \quad (5)$$

$$\sum_{l \in N} Y_{ijlmq} \leq X_m \quad \forall i \in I, j \in J, m \in N, q \in Q \quad (6)$$

$$\varsigma_{ijlm} Y_{ijlmq} \leq w_{ijq} (1 - V_{ijq}) (1 - \gamma) \quad \forall i \in I, j \in J, l \in N, m \in N, q \in Q \quad (7)$$

$$X_l \in \{0, 1\} \quad \forall l \in N \quad (8)$$

$$Y_{ijlmq} \in \{0, 1\} \quad \forall i \in I, j \in J, l \in N, m \in N, q \in Q \quad (9)$$

$$V_{ijq} \in \{0, 1\} \quad \forall i \in I, j \in J, q \in Q \quad (10)$$

Constraint 3 represents the number of candidate hybrid collaborative consolidation hubs to be located. Constraint 4 ensures that each origin–destination pair  $(i, j)$  and each collaborative carrier either must be assigned to exactly one hub pair or may not participate in the collaboration and transport its goods via its usual route. Because  $l$  may equal  $m$  under this constraint, it is possible that the shipment between origin–destination pair  $(i, j)$  may go through only a single hub. Constraints 5 and 6 state that shipments from origin  $i \in I$  to destination  $j \in J$  cannot be assigned to a hub at location  $l \in N$  or  $m \in N$  unless a hybrid collaborative consolidation hub is located at these candidate sites. Constraint 7 states that a specific carrier  $q \in Q$  for each origin–destination pair  $(i, j)$  will participate in the collaboration only if its routing costs are lower than the stand-alone costs by a prespecified margin. In Constraint 7,  $\gamma$  represents the profit margin expected by a collaborative carrier  $q \in Q$  to participate in the collaboration. Constraints 8, 9, and 10 represent the 0-1 integrality conditions for the decision variables  $(i, j)$ .

### Objective Function

$$\min \sum_{i \in I} \sum_{j \in J} \sum_{l \in N} \sum_{m \in N} \sum_{q \in Q} \zeta_{ijlm} d_{ijq} Y_{ijlmq} + \sum_{i \in I} \sum_{j \in J} \sum_{q \in Q} w_{ijq} d_{ijq} V_{ijq} + \sum_{l \in N} \sum_{q \in Q} \phi_{lq} X_l \quad (11)$$

The objective function seeks a set of candidate hybrid collaborative consolidation hubs to minimize the total transportation collaborative costs in a supply chain. It consists of three terms: the first term represents the total transportation costs associated with the carrier collaborative, the second represents the total costs associated with carriers not collaborating and shipping directly, and the third represents the total carrier collaborative costs associated with locating collaborative candidate hybrid consolidation facilities. The collaborative transportation costs are obtained as the summation of the product of the cost of travel for a shipment  $\zeta_{ijlm}$ , the collaborative carrier demand  $d_{ijq}$ , and  $Y_{ijlm}$  (the decision on whether a shipment travels via the collaborative hubs). Noncollaborative costs are obtained as the summation of the cost of shipping directly  $w_{ijq}$ , the collaborative carrier demand  $d_{ijq}$ , and  $V_{ijq}$  (the decision on whether to ship directly). The costs of a collaborative candidate hybrid consolidation hub location are obtained as the summation of the product of the costs of locating a collaborative hub  $\phi_{lq}$  and  $X_l$  (the decision on where a collaborative facility is located). Equation 11, subject to Constraints 3 through 10, represents the mathematical formulation of CCCMLP.

### Properties

The mathematical programming formulation of the CCCMLP belongs to the class of P-hub median location problems (18, 19, 32, 33). This is the case because Constraints 3, 4, 5, and 6 without  $V_{ijq}$  (the decision on whether to ship directly) reduces to a P-hub median problem. This class of problems is found to be NP-hard as the network and number of hubs increase (i.e., for  $p > 2$ ) (32, 34). Hence, the proposed solution is based on Lagrangian relaxation (32).

### Solution Method

The number of binary variables and constraints in the integer program explodes with an increase in problem size. For example, in a 10-node network, the number of binary integer variables is of the order of  $10^4$ . Therefore, the number of variables could become too large for regular solvers for problems of reasonable size. A heuristic based on Lagrangian relaxation is thus used to solve the model. In the preceding formulation, Constraints 5 and 6 are relaxed with  $\alpha_{ijlq}$ ,  $\forall i \in N, j \in N, l \in N, q \in Q$ , and  $\beta_{ijmq}$ ,  $\forall i \in N, j \in N, l \in N, q \in Q$  being the corresponding nonnegative Lagrange multipliers. For any specific value of  $(\alpha, \beta)$ , where  $\alpha = (\alpha_{ijlq}, \forall i \in N, j \in N, l \in N, q \in Q)$  and  $\beta = (\beta_{ijmq}, \forall i \in N, j \in N, l \in N, q \in Q)$ , the corresponding Lagrangian relaxed problem is

$$Z(\alpha, \beta) = \min \sum_{i \in I} \sum_{j \in J} \sum_{l \in N} \sum_{m \in N} \sum_{q \in Q} \bar{C}_{ijlmq} Y_{ijlmq} + \sum_{i \in I} \sum_{j \in J} \sum_{q \in Q} w_{ijq} d_{ijq} V_{ijq} + \sum_{l \in N} \bar{F}_l X_l$$

subject to Constraints 3, 4, 7, 8, 9, and 10

where

$$\bar{C}_{ijlmq} = \zeta_{ijlm} d_{ijq} + \alpha_{ijlq} + \beta_{ijmq} \quad \forall i \in N, j \in N, l \in N, m \in N, q \in Q$$

$$\bar{F}_l = \sum_{q \in Q} \phi_{lq} - \sum_{i \in I} \sum_{j \in J} \sum_{q \in Q} \alpha_{ijlq} - \sum_{i \in I} \sum_{j \in J} \sum_{q \in Q} \beta_{ijlq} \quad \forall l \in N$$

This formulation can be decomposed into two subproblems, SUB-I and SUB-II:

$$Z_{R1}(\alpha, \beta) = \min \sum_{i \in I} \sum_{j \in J} \sum_{l \in N} \sum_{m \in N} \sum_{q \in Q} \bar{C}_{ijlmq} Y_{ijlmq} + \sum_{i \in I} \sum_{j \in J} \sum_{q \in Q} w_{ijq} d_{ijq} V_{ijq} \quad (SUB-I)$$

subject to Constraints 4, 7, 9, and 10 and

$$Z_{R2}(\alpha, \beta) = \min \sum_{l \in N} \bar{F}_l X_l \quad (SUB-II)$$

subject to Constraints 3 and 8.

Subproblems SUB-I and SUB-II can be solved with the following methods. Let  $\underline{Y}_{ijlmq}$ ,  $\underline{V}_{ijq}$ , and  $\underline{X}_l$  denote the solution to SUB-I and SUB-II.

### Solving SUB-I

SUB-I decomposes into  $|N|^2|Q|$  minimization problems for each  $(i \in N, j \in N, q \in Q)$  tuple.

1. The first step in solving SUB-I is prescreening for variables that do not satisfy Constraint 7. Thus for every  $i \in I, j \in J, l \in N, m \in N, q \in Q$ , if  $\zeta_{ijlm} > w_{ijq} (1 - \gamma)$ , then  $\underline{Y}_{ijlmq}$  can be fixed to zero. This is because it is always more beneficial for the carrier to send goods directly than through hubs  $l$  and  $m$ .

2. In the second step, for each  $i \in I, j \in J, q \in Q$  the minimum value  $\bar{C}_{ijq}^{min} = \min \{\bar{C}_{ijlmq} : l \in N, m \in N \text{ such that } \underline{Y}_{ijlmq} \neq 0\}$  is found. Let  $l'$  and  $m'$  be the hub locations on the route corresponding to  $\bar{C}_{ijq}^{min}$ . If  $\bar{C}_{ijq}^{min} \leq w_{ijq} d_{ijq}$ , then set  $\underline{Y}_{ijl'm'q} = 1, \underline{V}_{ijq} = 0$ ; otherwise set  $\underline{Y}_{ijl'm'q} = 0, \underline{V}_{ijq} = 1$ .

### Solving SUB-II

SUB-II can be solved by sorting the values of  $\bar{F}_l$  in ascending order, choosing the  $p$  minimum values, and setting the corresponding  $\underline{X}_l$  to be equal to 1.

The solution obtained from the preceding procedures by solving SUB-I and SUB-II may not be feasible for the original formulation because Constraints 5 and 6 may not be satisfied, that is, carriers might be routing goods through unopened hubs. For any set of values for  $\alpha$  and  $\beta$ , the solution  $(\underline{Y}_{ijlmq}, \underline{V}_{ijq}, \text{ and } \underline{X}_l, \forall i \in N, j \in N, l \in N, m \in N, q \in Q)$  can be used to find a lower bound to the original problem as  $Z_{R1}(\alpha, \beta) + Z_{R2}(\alpha, \beta) \leq Z(\alpha, \beta)$ .

The solution obtained from solving SUB-I and SUB-II can be converted to a feasible solution  $(\bar{Y}_{ijlmq}, \bar{V}_{ijq}, \text{ and } \bar{X}_l, \forall i \in N, j \in N, l \in N, m \in N, q \in Q)$  with the procedure given next.



### Obtaining Feasible Solution

For each  $(i \in N, j \in N, q \in Q)$  tuple, if  $\underline{V}_{ijq} = 1$ , the carrier is sending goods directly and the route is feasible, and set the corresponding  $\bar{V}_{ijq} = 1$ . If  $\underline{V}_{ijq} = 0$ , then find the hub locations  $l'$  and  $m'$  through which the carrier routes the goods. Depending on the values of  $\underline{X}_{l'}$  and  $\underline{X}_{m'}$ , four cases are possible:

Case 1. If  $\underline{X}_{l'} = 1$  and  $\underline{X}_{m'} = 1$ , then the carrier is routing the goods through opened hubs and therefore the route is feasible. Set  $\bar{Y}_{ijl'm'q} = 1$ .

Case 2. If  $\underline{X}_{l'} = 1$  and  $\underline{X}_{m'} = 0$ , then the carrier is routing goods through unopened hub  $m'$ . In this case, determine  $\bar{C}_{ijl'm'q}^{\min} = \min\{\bar{C}_{ijl'm'q} : m \in N \text{ such that } \underline{Y}_{ijl'm'q} \neq 0 \text{ and } \underline{X}_m = 1\}$  and let  $m''$  be the hub corresponding to  $\bar{C}_{ijl'm'q}^{\min}$ . Compare the modified costs of all routes  $(i - l' - m - j)$  with open hubs  $\{m \in N : \underline{X}_m = 1\}$  that have not been fixed to 0 in the prescreening stage,  $\underline{Y}_{ijl'm'q} \neq 0$ , and find the minimum cost route. Then set the corresponding  $\bar{Y}_{ijl'm''q} = 1$ . If there are no routes  $(i - l' - m - j)$  with open hubs  $\{m \in N : \underline{X}_m = 1\}$  that have not been fixed to 0 in the prescreening stage, then set the corresponding  $\bar{V}_{ijq} = 1$ .

Case 3. If  $\underline{X}_{l'} = 0$  and  $\underline{X}_{m'} = 1$ , then the carrier is routing goods through unopened hub  $l'$ . In this case, determine  $\bar{C}_{ijl'm'q}^{\min} = \min\{\bar{C}_{ijl'm'q} : l \in N \text{ such that } \underline{Y}_{ijl'm'q} \neq 0 \text{ and } \underline{X}_l = 1\}$  and let  $l''$  be the hub corresponding to  $\bar{C}_{ijl'm'q}^{\min}$ . Compare the modified costs of all routes  $(i - l - m' - j)$  with open hubs  $\{l \in N : \underline{X}_l = 1\}$  that have not been fixed to 0 in the prescreening stage,  $\underline{Y}_{ijl'm'q} \neq 0$ , and find the minimum cost route. Then set the corresponding  $\bar{Y}_{ijl''m'q} = 1$ . If there are no routes  $(i - l - m' - j)$  with open hubs  $l \in N$  that have not been fixed to 0 in the prescreening stage, then set the corresponding  $\bar{V}_{ijq} = 1$ .

Case 4. If  $\underline{X}_{l'} = 0$  and  $\underline{X}_{m'} = 0$ , then the carrier is routing goods through unopened hub  $l'$  and  $m'$ . In this case determine  $\bar{C}_{ijl'm'q}^{\min} = \min\{\bar{C}_{ijl'm'q} : l \in N, m \in N \text{ such that } \underline{Y}_{ijl'm'q} \neq 0 \text{ and } \underline{X}_l = 1 \text{ and } \underline{X}_m = 1\}$  and let  $l'', m''$  be the hubs corresponding to  $\bar{C}_{ijl'm'q}^{\min}$ . Compare the modified costs of all routes  $(i - l - m - j)$  with open hubs  $\{l, m \in N : \underline{X}_m = 1 \text{ and } \underline{X}_l = 1\}$  that have not been fixed to 0,  $\underline{Y}_{ijl'm'q} \neq 0$  in the prescreening stage and find the minimum cost route. Then set the corresponding  $\bar{Y}_{ijl''m''q} = 1$ . If there are no routes  $(i - l - m - j)$  with open hubs  $l, m \in N$  that have not been fixed to 0 in the prescreening stage, then set the corresponding  $\bar{V}_{ijq} = 1$ .

### Lagrangian Relaxation Procedure

The Lagrangian relaxation procedure can be summarized in the following steps:

Step 1. Initialization. Set  $\alpha_{ijlq} = 0, \beta_{ijlq} = 0, \forall i \in N, j \in N, l \in N, q \in Q$ . Set the value of the current best upper bound,  $UB = \infty$  and the current best lower bound  $LB = -\infty$ . Set  $\Delta = 2$ .

Step 2. Lower bound. Solve SUB-I and SUB-II for current values of  $\alpha_{ijlq}, \beta_{ijlq}, \forall i \in N, j \in N, l \in N, q \in Q$  and determine  $Z_{LB}(\alpha, \beta)$ . If  $Z_{LB}(\alpha, \beta) > LB$ , then update  $LB = Z_{LB}(\alpha, \beta)$ .

Step 3. Upper bound. Transform the lower bound solution  $(\underline{Y}, \underline{V}, \underline{X})$  to a feasible solution  $(\bar{Y}, \bar{X}, \bar{V})$  by using the preceding procedure to determine a feasible upper bound  $Z_{UB}$ . If  $Z_{UB} < UB$ , then update  $UB = Z_{UB}$ .

Step 4. Updating Lagrange multipliers. Update the multipliers based on the lower bound solution as follows:

$$s_{ijlq} = \sum_{m \in N} \underline{Y}_{ijlmq} - \underline{X}_l \quad \forall i \in I, j \in J, l \in N, q \in Q$$

$$r_{ijmq} = \sum_{l \in N} \underline{Y}_{ijlmq} - \underline{X}_m \quad \forall i \in I, j \in J, m \in N, q \in Q$$

$$t = \Delta \frac{[Z_{UB} - Z_{LB}(\alpha, \beta)]}{\left(\sum_{i \in I} \sum_{j \in J} \sum_{l \in N} \sum_{q \in Q} s_{ijlq}\right)^2 + \left(\sum_{i \in I} \sum_{j \in J} \sum_{m \in N} \sum_{q \in Q} r_{ijmq}\right)^2}$$

$$\alpha_{ijlq}^+ = \alpha_{ijlq} + t \left( \sum_{m \in N} \underline{Y}_{ijlmq} - \underline{X}_l \right) \quad \forall i \in I, j \in J, l \in N, q \in Q$$

$$\beta_{ijlq}^+ = \beta_{ijlq} + t \left( \sum_{m \in N} \underline{Y}_{ijlmq} - \underline{X}_m \right) \quad \forall i \in I, j \in J, l \in N, q \in Q$$

The parameter  $\Delta$  is halved if there are no updates in the lower bound objective function for 10 iterations. If the upper bound is updated,  $\Delta$  is reset to 2.

Step 5. Convergence. The algorithm converges if any of the following three conditions is satisfied: (a) the number of iterations is equal to a prespecified maximum number of iterations (1,500); (b)  $\Delta$  becomes less than a prespecified minimum value (0.0025); and (c) there is no improvement in the upper bound for a fixed number of iterations (200). If the algorithm has not converged, go to Step 2.

## STUDY EXPERIMENTS

The study experiments analyze the performance of the CCCMLP under an individual rate-setting behavioral strategy. (All carriers in the collaborative system assume the same rate-setting behavior.) In addition, varying degrees of  $\gamma$ , the profit margin expected by a carrier to participate in the collaboration, are studied. This is done to determine the point at which collaborating is cost-effective to carriers considering the various network sizes.

### Data Generation and Implementation of Solution Method

The data for the CCCMLP problem were simulated with a uniform distribution of industry ranges and values (produced 10 sets of data and used the average values) introduced by Hernandez and Peeta for (a) the revenue-oriented rate-setting behavior, (b) the costs of establishing a hybrid collaborative consolidation candidate transshipment hub, (c) the origin–destination demand for multiple shipments, and (d) the collaborative costs (6). A diesel fuel price of \$3.79 per gallon is assumed.

The CCCMLP was coded in C++ in a standard compiler and computing environment consisting of a Dell T710 machine with Intel Xeon X5680 with Windows 7 Enterprise 64-bit operating with 3.33 GHz and 8 GB RAM.

### Experiment Setup

The setup for the experiment consists of three collaborating carriers for the CCCMLP problem. The additional problem parameters take values according to the following ranges: network size in number of nodes (10 and 20) and number of hubs (two, three, four, and five). MATLAB was used to randomly generate the 10-node network

and the 20-node network. As the data are simulated, 10 randomly generated data sets consistent with the observed ranges for small to medium-size LTL industries are created and averaged to create a single data set. For each scenario of network size and number of hubs, the collaborative rates, noncollaborative costs, and location costs are randomly generated in addition to the demand.

## ANALYSIS OF RESULTS

The CCCMLP is addressed in a static planning context, and insight can be obtained into how varying degrees of expected profit margins affect the centralized carrier collaborative network. From the central entity's perspective, selection of the hybrid collaborative consolidation candidate transshipment hubs can be made only if the collaborative routing and hub location costs coupled with the direct route costs (noncollaborative costs) for the system are minimized.

The potential for collaboration among carriers is investigated by focusing on the level of monetary savings due to expected profit margins (see Equation 7). These margins are reflected through the parameter  $\gamma$ , which takes the values 9%, 18%, 36%, 48%, 60%, 72%, 84%, and 96%. In general, a lower profit margin value leads to greater levels of collaboration. As seen from Equation 7,  $(1 - \gamma)$  is the true profit margin, so the lower the  $\gamma$ , the greater the profit margin.

The values were chosen arbitrarily in increments of 9% to capture changes in the total savings. Tables 1 and 2 show the results of the analysis.

Tables 1 and 2 provide a comparison of the number of hubs and total savings for changes in  $\gamma$  for a 10-node and a 20-node network, respectively. The total savings represent the cost differences between noncollaborative (or direct route) and collaborative (collaborative routes) as a percentage. The overall trend of the results for each number-of-hubs scenario and network size indicates that as the  $\gamma$  increases, the number of direct routes increases. This shows the sensitivity of carrier shipments (Column 4) to changes in expected profits. Similarly, the lower the  $\gamma$ , the greater the percentage of collaborated routes and total percentage of savings (Columns 6 and 7). The collaborative hubs selected for each scenario are shown in Tables 1 and 2, respectively. As profit margins decreased, direct routes became the preferred route for the carriers; however, hubs were still selected because the formulation did not preclude their selection. The selected hubs indicate the locations that would best facilitate the collaboration. In a future extension of the work, Equation 3 will be omitted and a corresponding weight used for costs for collaborative candidate hybrid consolidation hub locations in the objective function to analyze trade-offs with other objective cost components.

Tables 1 and 2 do not include computational times, because on average, the Lagrangian relaxation solution method presented here solved the majority of scenarios within 5 min, and no other algorithm

**TABLE 1 Ten-Node Network with Three Collaborative Carriers**

Number of Hubs ( $P \geq 2$ )	$\gamma$ Value (profit margin) (%)	Selected Collaborative Hubs	Number of Direct Routes	Number of Collaborative Routes	Routes Collaborated (%)	Total Savings (%)
2	9.00	1, 4	22	248	91	43
	18.00	1, 4	45	225	83	42
	36.00	1, 4	95	175	64	37
	48.00	1, 4	152	118	43	28
	60.00	1, 4	198	72	26	18
	72.00	1, 4	246	24	8	6
	84.00	1, 4	264	6	2	1
	96.00	1, 4	270	0	0	0
3	9.00	1, 4, 9	12	258	95	48
	18.00	1, 4, 9	24	246	91	47
	36.00	1, 4, 9	61	209	77	44
	48.00	1, 4, 9	108	162	60	36
	60.00	1, 4, 9	164	106	39	25
	72.00	1, 4, 9	223	47	17	10
	84.00	1, 4, 9	252	18	6	1
	96.00	1, 4, 9	270	0	0	0
4	9.00	1, 4, 7, 9	8	262	97	52
	18.00	1, 4, 7, 9	18	252	93	51
	36.00	1, 4, 7, 9	39	231	85	50
	48.00	1, 4, 7, 9	68	202	74	46
	60.00	1, 4, 7, 9	124	146	54	35
	72.00	1, 4, 7, 9	195	75	27	16
	84.00	1, 4, 7, 9	234	36	13	3
	96.00	1, 4, 7, 9	270	0	0	0
5	9.00	1, 2, 4, 7, 9	4	266	98	54
	18.00	1, 2, 4, 7, 9	11	259	95	53
	36.00	1, 2, 4, 7, 9	28	242	89	52
	48.00	1, 2, 4, 7, 9	50	220	81	49
	60.00	1, 2, 4, 7, 9	99	171	63	40
	72.00	1, 2, 4, 7, 9	168	102	37	23
	84.00	1, 2, 4, 7, 9	210	60	22	8
	96.00	1, 2, 4, 7, 9	270	0	0	0

TABLE 2 Twenty-Node Network with Three Collaborative Carriers

Number of Hubs ( $P \geq 2$ )	$\gamma$ Value (profit margin) (%)	Selected Collaborative Hubs	Number of Direct Routes	Number of Collaborative Routes	Routes Collaborated (%)	Total Savings (%)
2	9.00	7, 15	121	1,019	89	76
	18.00	7, 15	222	918	80	71
	36.00	7, 15	507	633	55	53
	48.00	7, 15	798	342	31	29
	60.00	7, 15	997	143	12	13
	72.00	7, 15	1,092	48	4	5
	84.00	7, 15	1,134	6	0	0
	96.00	7, 15	1,140	0	0	0
3	9.00	7, 11, 15	82	1,058	92	80
	18.00	7, 11, 15	169	971	85	75
	36.00	7, 11, 15	418	722	63	60
	48.00	7, 11, 15	686	454	39	38
	60.00	7, 11, 15	917	223	19	19
	72.00	7, 11, 15	1,057	83	7	8
	84.00	7, 11, 15	1,122	18	1	2
	96.00	7, 11, 15	1,140	0	0	0
4	9.00	7, 11, 12, 15	65	1,075	94	81
	18.00	7, 11, 12, 15	138	1,002	87	78
	36.00	7, 11, 12, 15	359	781	68	64
	48.00	7, 11, 12, 15	591	549	48	46
	60.00	7, 11, 12, 15	836	304	26	27
	72.00	7, 11, 12, 15	1,026	114	10	11
	84.00	7, 11, 12, 15	1,104	36	3	3
	96.00	7, 11, 12, 15	1,140	0	0	0
5	9.00	7, 11, 12, 14, 15	46	1,094	95	83
	18.00	7, 11, 12, 14, 15	109	1,031	90	80
	36.00	7, 11, 12, 14, 15	284	856	75	69
	48.00	7, 11, 12, 14, 15	505	635	55	53
	60.00	7, 11, 12, 14, 15	770	370	32	32
	72.00	7, 11, 12, 14, 15	983	157	13	15
	84.00	7, 11, 12, 14, 15	1,080	60	5	5
	96.00	7, 11, 12, 14, 15	1,140	0	0	0

was considered for solving the CCCMLP. An attempt was made to model the CCCMLP with CPLEX; however, the exact method (the branch-and-cut algorithm) could not solve to optimality network sizes that were greater than five nodes in a reasonable amount of time (the time exceeded 3,600 s).

Figure 1 is a comparison of  $\gamma$  with the percentage of total savings for various numbers of collaborative hubs ( $P$ ). As shown in the figure, for a greater number of hubs, the net gains start to level off. This indicates that if the costs of the hybrid consolidation hub locations remain constant, the overall savings difference in route collaboration and direct route at the lower-level margins is limited to magnitude of the location costs. This is especially true for the smaller network sizes (Figure 1a); still, for the 20-node network (Figure 1b), a larger percentage savings is observed. This is the case because more carrier shipments need to be made, an indication that the costs associated with the collaboration and direct routes outweigh the costs to place a collaborative candidate hybrid consolidation hub.

The study experiments provide insight into the various expected profit margins and their ability to induce collaboration in the centralized carrier collaborative network to form a hybrid hub-and-spoke system. The sensitivity results indicate that at higher levels of expected profit margins, the carriers are more apt to participate in the location of candidate hybrid consolidation hubs and the routing of collaborative shipments.

## SUMMARY AND FUTURE WORK

A CCCMLP was introduced that provides a planning framework for analyzing the benefits of a centralized multiple-carrier collaborative network for the creation of a hybrid hub-and-spoke system. The problem addresses operational issues related to transfer locations and shipment consolidation by introducing the concept of hybrid consolidation hubs from existing locations without the need to construct or invest new consolidation facility infrastructure. This is done by leveraging the current service locations of existing LTL collaborative carriers and using novel opportunities provided through advances in ICT and e-commerce. An uncapacitated P-hub median location mathematical programming formulation was presented for a rate-setting behavioral strategy for the collaborative system. The corresponding formulation was shown to be NP-hard and was solved with a Lagrangian relaxation approach.

The study results indicate that larger expected profit margins from the collaborative carriers with revenue-generating behavior would increase the likelihood that carriers will collaborate. In addition, as the network size increases, the effect of hybrid hub location costs was smaller. A key inference of this study is that carrier collaboration through a collaborative hybrid hub-and-spoke system can be a critical strategy to allow small to medium-sized LTL carriers to remain competitive, by decreasing their operational costs for shipments across a point-to-point network.

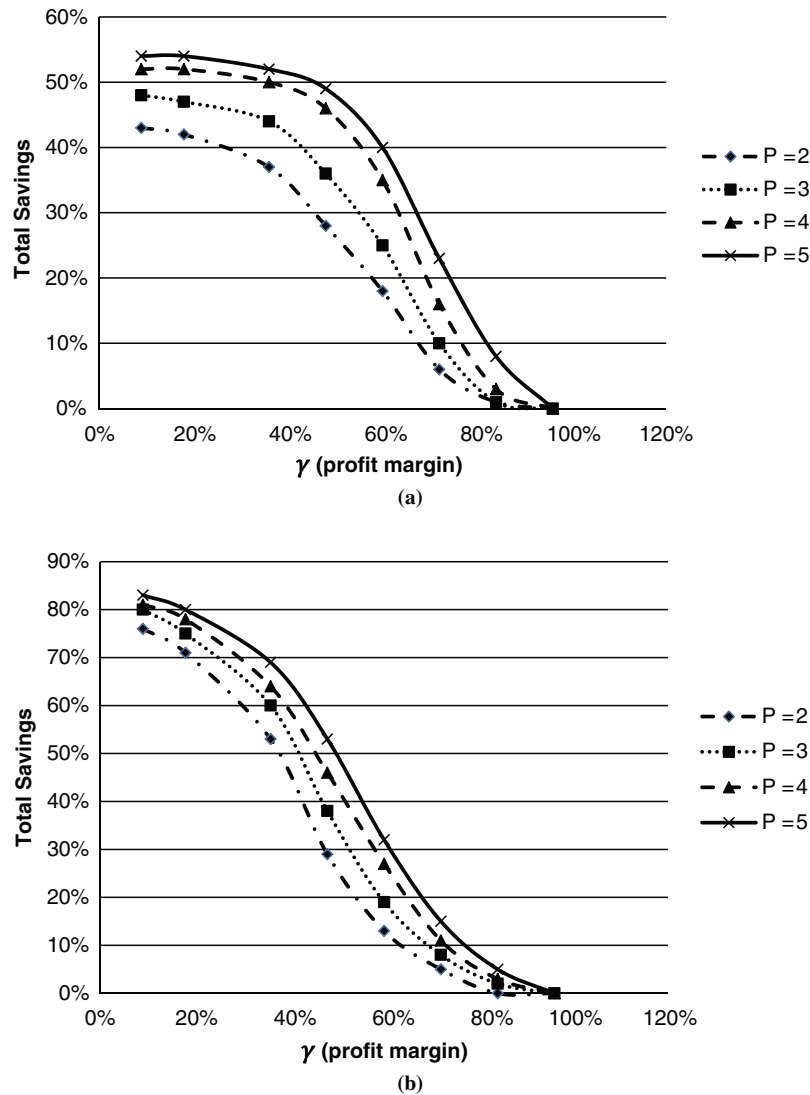


FIGURE 1 Comparison of  $\gamma$  to total savings for varying number of collaborative hubs ( $P$ ) for (a) 10-node network and (b) 20-node network.

Future work will address various rate-setting behaviors and study the effect of centralized carrier costs on the number of hubs selected (i.e., not predefining the number of hubs). In addition, facility capacities will be incorporated, increasing the complexity of the problem.

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