Heavy-Vehicle Crash Rate Analysis

Comparison of Heterogeneity Methods Using Idaho Crash Data

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Studies investigating crash rates by roadway classification are few and far between and even more rare if extended to focus on heavy vehicles. This study explored and compared two advanced econometric methodsrandom-parameter Tobit regression and latent class Tobit regressionto determine contributing factors for heavy-vehicle crashes per million vehicle miles traveled while accounting for the unobserved heterogeneity present in crash data. The increasing crash rates in Idaho, crash proportion by roadway classification, and available data made an ideal case study. Empirical results show that although the random-parameter Tobit regression model provides better insight into heavy-vehicle crash rates than the fixed-parameter approach, the latent class Tobit regression model is the preferred methodology for the given data set. Traffic volumes, roadway characteristics, and traffic control devices were among the variables found to be statistically significant. Results from this study provide an alternate framework to account for heterogeneity while identifying key factors by roadway classification that influence heavy-vehicle crash rates. The illustrated framework and analysis by roadway classification can provide guidance to transportation agencies and policy makers and prompt future studies to include a latent class analysis, analysis by road classification, or both.

Heavy-vehicle crashes have a substantial economic impact on commerce and society. In the United States, heavy-vehicle crashes cost about \$87 billion in 2011 and costs due to delay and other consequences were roughly \$28 billion (1, 2). These values will continue to increase as the economy continues to grow, as will the volume of heavy vehicles on the nation's freight infrastructure. For example, from 2010 to 2013 a 2.3% increase in heavy vehicles was experienced (about 6,000,000 vehicles) (3). This number is expected to continue to grow and crashes associated with heavy vehicles will remain a concern for safety planners and safety-related agencies. Although heavy-vehicle crashes have decreased over the past two decades, the number of fatal crashes per 100 million vehicle miles traveled (MVMT) compared with passenger cars is higher (1.34 versus 1.08 in 2014) (4, 5). In Idaho, the state experienced a 5.6% increase in heavy-vehicle crashes and a 4.4% increase in heavy-vehicle crashes per MVMT from 2010 to 2013 (6). Fifty percent of these crashes occurred on local roads, 28% of injury crashes happened on Interstates, and approximately 68% of fatalities happened on U.S. and state highways (6). These statistics illustrate the need for continued research in understanding the relationship between heavy-vehicle crash rates and roadway classification.

A number of studies have addressed crash frequency through the application of count- and spatial-based models (7–15). However, most of these studies have focused on data related to pedestrians, passenger cars, or all traffic mixes in a single modeling framework and do not address heavy vehicles explicitly. Although there have been several recent efforts to understand heavy-vehicle injury severity factors (16–18), heavy-vehicle crash rate analyses are sparse. This lack is especially true for heavy-vehicle crash rates by functional class of road. A possible reason for this deficiency in the literature may stem from the availability of sufficient data to capture the complex interactions of multiple crash rate factors under a single framework by functional class.

Recent studies have addressed the issue of insufficient data through the application of statistical and econometric methods that account for unobserved factors (unobserved heterogeneity), which are factors unknown to the analyst and which may vary across observations; a complete review of these methods may be found elsewhere (19). For instance, weather conditions continually change over time, as well as driver response to the changing weather conditions. These models allow the analyst to account for these variations and make more informed inferences regarding the effects of the contributing factors (19).

With this aspect in mind, the current study seeks to identify factors that affect heavy-vehicle crashes per MVMT by road classification through the application and performance-based comparison of two "heterogeneity" models, namely, random-parameter and latent class Tobit regression. The Tobit modeling framework is selected because of the nature of crash rate data. Similar to frequency models, a crash rate analysis is likely to have several observations in which no crash has occurred, and therefore a censoring method is recommended to account for the skewed nature of the response variable (crash rate). It has been shown that the Tobit regression framework can account for the skewed nature of crash rate data without omitting observations by censoring the analysis at a given value (20). These models have been successfully applied to related transportation safety data; for example, Anastasopoulos et al. used the fixed-parameter Tobit model to investigate crash rates on Interstates in Indiana and determine contributing factors (21). To extend the Tobit framework, Anastasopoulos et al. utilized the random-parameter Tobit model to determine factors that influence crash rates per 100 MVMT on highways (22). Bin Islam and Hernandez investigated fatalities per million truck miles traveled and fatalities per ton-mile of freight for heavy vehicles through the application of a random-parameter Tobit

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regression model (23), and Chen et al. utilized a random-effects Tobit model to analyze crash rates with refined-scale data (24). From a latent class Tobit regression application, there are no known applications to transportation safety data; however, the method has been applied to social science studies [studies by Jedidi et al. (25)and Brown et al. (26)].

Therefore, the current study will use the random-parameter Tobit method to identify significant contributing factors to crash rates by roadway functional class while accounting for heterogeneity. However, variables not found to be random in the random-parameter method may, in fact, have varying effects on heavy-vehicle crash rates. Hence, the current study will be extended by investigating the results of the Tobit latent class approach by disaggregating the Tobit model into unobserved groups (or classes). An extensive crash database collected and maintained by the Idaho Department of Transportation is used. The findings of this study can provide insight that can aid safety planners and safety-related agencies in identifying appropriate countermeasures to help reduce and mitigate heavy-vehicle crashes. To the best of the authors' knowledge, these are the first attempts at developing these types of models for heavy-vehicle crash rate analysis.

SOURCE OF DATA

The current study used 7 years of police-recorded crash data obtained from the State of Idaho (2007 to 2013). Each year was filtered to represent heavy-vehicle crashes and then combined with traffic data from the Idaho Department of Transportation utilizing segment codes and milepost markers that were present in both data sets. The segment codes and milepost numbers of the location of the crash were used to determine the intermediate segments within the milepost intervals in the traffic data; these segments are used for the modeling process. Using the complete data set consisting of exposure variables—roadway geometrics, traffic control devices, number of lanes, and so forth—and traffic volumes, several indicator variables were created to identify specific exposure conditions and traffic volumes that affect crash rates by road classification in Idaho. The final data set for principal arterials, major collectors, and Interstates had 1,560, 1,010, and 1,588 heavy-vehicle crashes, respectively. Table 1 displays the response variable and indicators found to be significant throughout the modeling process.

METHODOLOGY

Dependent Variable

To model heavy-vehicle crash rates, a rate for each segment is calculated by using the traffic data provided by Idaho (27):

$$R_{s} = \frac{\sum_{y=1}^{n} N_{ys}}{\left[\sum_{y=1}^{n} AADT_{ys} \times L_{s} \times 365\right]}$$
(1)

1,000,000

where

 R_s = number of crashes per MVMT on segment *s*, y = year (2007 to 2013),

 N_{ys} = number of heavy-vehicle crashes in year y on segment s, AADT_{ys} = average annual daily traffic for year y on segment s, and

 L_s = length of segment *s* (mi).

Given the data sets available for the current study, a specific methodology was employed to get corresponding traffic data for each crash, as described earlier. Since the current study used 7 years of crash data, the likelihood of having one crash on each segment

TABLE 1 Descriptive Statistics for Significant Variables by Road Classification

Classification	Variable	Mean	Standard Deviation	Min.	Max.
Principal	Crashes per MVMT (response variable)	0.127	0.208	0.005	3.488
arterials	Speed limit (1 if 65 mph, 0 otherwise)	0.443	0.497	na	na
	Traffic control device (1 if no device, 0 otherwise)	0.702	0.458	na	na
	Road configuration (1 if 2-way and 2-way left-turn lane)	0.126	0.333	na	na
	Heavy-vehicle AADT (1 if less than or equal to 300, 0 otherwise)	0.158	0.365	na	na
	Passenger vehicle AADT (1 if greater than 10,500, 0 otherwise)	0.108	0.310	na	na
	Total AADT (1 if between 5,000 and 7,000, 0 otherwise)	0.167	0.373	na	na
Major collectors	Crashes per MVMT (response variable)	0.620	1.204	0.017	13.105
	Speed limit (1 if less than or equal to 40 mph, 0 otherwise)	0.241	0.428	na	na
	Traffic control device (1 if stop sign, 0 otherwise)	0.159	0.366	na	na
	Horizontal geometrics (1 if straight, 0 otherwise)	0.789	0.408	na	na
	Road configuration (2-way and double-yellow painted divider, 0 otherwise)	0.129	0.335	na	na
	Passenger vehicle AADT (1 if greater than 2,500, 0 otherwise)	0.227	0.419	na	na
	Total AADT (1 if less than 500, 0 otherwise)	0.228	0.420	na	na
Interstates	Crashes per MVMT (response variable)	0.034	0.047	0.003	0.726
	Speed limit (1 if 75 mph, 0 otherwise)	0.675	0.469	na	na
	Road configuration (1 if 2-way and raised or depressed divider)	0.929	0.258	na	na
	Heavy-vehicle AADT (1 if between 2,000 and 3,000, 0 otherwise)	0.256	0.437	na	na
	Horizontal geometrics (1 if curved, 0 otherwise)	0.230	0.421	na	na
	Surface defects (1 if no surface defects, 0 otherwise)	0.960	0.195	na	na
	Passenger vehicle AADT (1 if greater than 15,000, 0 otherwise)	0.116	0.321	na	na
	Total AADT (1 when less than 6,500, 0 otherwise)	0.164	0.370	na	na

NOTE: na = not applicable; AADT = average annual daily traffic.

was higher as compared with using a 3- to 5-year period. As such, many segments for each road classification were identified and analyzed, each of which had at least one crash. Referring to Table 1, the minimum values of the response variables are approximately zero; if fewer years of crash data were studied, these values could have very likely been zero. Figure 1 illustrates the crash rate distribution for each road classification in which the skewed distributions needed to be addressed during analysis.

Random-Parameter Tobit Model

The distribution of crash rates illustrates the need to utilize a method that can account for the large lower-bound cluster of observations while maintaining the linear assumptions required for regression of a continuous dependent variable (heavy-vehicle crash rates by roadway classification). With regard to other principal arterials, the data were too centered at zero and, even with censoring, produced erroneous estimates during analysis; therefore, this classification was omitted from the study. Taking this into consideration, the current study sought to develop a statistical model that could be used to determine the contributing factors on heavy-vehicle crash rates by roadway classification. This study applied the Tobit regression modeling framework (28). However, key variables not available within many crash data sets and variation across the available variables often

result in unobserved heterogeneity and, if neglected, will lead to biased estimates and inaccurate inferences [further discussion may be found elsewhere (19)]. To account for such problems the current study applied the random-parameter approach to the traditional Tobit regression framework (22, 23, 29–31). As mentioned earlier, Anastasopoulos et al. (22), Bin Islam and Hernandez (23), and Chen et al. (24) all applied the random-parameter Tobit regression model successfully to transportation safety data. Therefore, for this work, the standard Tobit model is expressed as follows:

$$Y_{s}^{*} = \boldsymbol{\beta}' \mathbf{X}_{s} + \boldsymbol{\varepsilon}_{s} \quad \text{with } \boldsymbol{\varepsilon}_{s} \sim N[0, \sigma^{2}] \text{ and } s = 1, 2, \dots, N$$
$$Y_{s} = \begin{cases} Y_{s}^{*} & \text{if } Y_{s}^{*} > L\\ 0 & \text{if } Y_{s}^{*} \leq L \end{cases}$$
(2)

where

- Y_s = number of crashes per MVMT,
- L = value at which model is left-censored,
- \mathbf{X}_s = vector of explanatory variables (AADT, roadway geometrics, and so forth),
- β' = vector of estimated parameters, and
- ε_s = normally and independently distributed error term with mean of zero and constant variance, σ^2 .



FIGURE 1 Heavy-vehicle crash rate distribution by roadway classification: (a) principal arterials, (b) major collectors, (c) Interstates, and (d) other principal arterials.

To determine the likelihood for the Tobit model over zero observations (e.g., the value at which the Tobit model is left-censored) and positive observations (Equation 1), the following function applies (21, 26):

$$L = \prod_{0} \left[1 - \Psi \left(\frac{\boldsymbol{\beta}' \mathbf{X}_s}{\sigma} \right) \right] \prod_{1} \left(\frac{1}{\sigma} \right) \Psi \left(\frac{Y_s - \boldsymbol{\beta}' \mathbf{X}_s}{\sigma} \right)$$
(3)

where

$$\Psi\left(\frac{\mathbf{\beta}'\mathbf{X}_s}{\sigma}\right)$$

is the standard normal distribution function and

$$\Psi\left(\frac{Y_s - \boldsymbol{\beta}' \mathbf{X}_s}{\sigma}\right)$$

is the standard normal density function.

In an attempt to capture the unobserved heterogeneity, the randomparameter approach is now applied to the Tobit framework and estimated parameters can be written as follows (*32*):

$$\beta_s = \beta + \phi_s \tag{4}$$

where the equivalent log likelihood function is (26)

$$\log \mathbf{L} = \sum_{\forall s} \ln \int_{\phi_s} g(\phi_s) P(Y_s^* | \phi_s) d\phi_s$$
(5)

where $g(\phi_s)$ is the probability density function of ϕ_s and $P(Y_s^* | \phi_s)$ is the probability of the Tobit model (i.e., probability that the value is uncensored). As stated in previous studies (22, 23), the maximum likelihood estimations encounter computing issues because of their complexity. To address this issue, a common approach developed by Halton is used to simulate the maximum likelihood by utilizing Halton draws to solve the complex integral seen in Equation 5, which has been shown to be preferable over merely random draws (33–35).

Latent Class Tobit Model

Although the random-parameter method accounts for unobserved heterogeneity, there is a possible disadvantage because of the assumption that the parameters vary in a predefined distribution and that parameters vary only across singular observations [further discussion may be found elsewhere (19)]. Taking that into consideration, the latent class approach attempts to capture unobserved heterogeneity by allowing estimable parameters to vary with an underlying discrete distribution across unobserved groups of observations (or classes). The heterogeneity is accounted for by defining a finite number of points and measuring the mass probability of the intervals between points. Applying this operation to the Tobit regression structure results in

$$Y_{s}^{*}|(\text{class} = C) = \boldsymbol{\beta}_{c}^{\prime} \mathbf{X}_{s} + \boldsymbol{\varepsilon}_{slc} \qquad \text{with } \boldsymbol{\varepsilon}_{slc} \sim N\left[0, \boldsymbol{\sigma}_{c}^{2}\right]$$
$$\text{and } s = 1, 2, \dots, N$$
$$Y_{s} = \begin{cases} Y_{s}^{*} & \text{if } Y_{s}^{*} > L\\ 0 & \text{if } Y_{s}^{*} \leq L \end{cases}$$
(6)

where β'_c is a vector of estimated parameters belonging to Class *C* and $Y^*_s|(\text{class} = C)$ is the number of crashes per MVMT of segment *s* in Class *C*. The corresponding log likelihood function is now as follows (26):

$$\log \mathbf{L} = \sum_{s=1}^{N} \log \left[\sum_{c=1}^{C} P_{sc}(\boldsymbol{\delta}_{c}, \boldsymbol{\omega}_{s}) \left[f\left(Y_{s} | \text{class} = C, \mathbf{X}_{s}, \boldsymbol{\beta}_{c}^{\prime} \boldsymbol{\sigma}_{c} \right) \right] \right]$$
(7)

where $P_{sc}(\delta_c, \omega_s)$ is the prior to model estimation logit probability of being in Class *C* and represented by the multinomial logit form (26):

$$P_{sc}(\delta_c, \omega_s) = \frac{e^{(\omega_s \delta_c)}}{\sum_{c=1}^{C} e^{(\omega_s \delta_c)}} \qquad \text{with } c = 1, 2, \dots, C$$
(8)
and $\delta_c = 0$ for normalization

Last, after the parameters have been estimated, a second estimation is conducted to determine the posterior probabilities of crash rate Y_s belonging to Class C (36). The posterior probability that a heavy-vehicle crash belongs to Class C is determined after the estimation. That is, the posterior probability utilizes the estimated parameters to determine a class probability based on the observed crash data (26, 37):

$$P(\text{class} = C | \text{crash rate } Y_s)$$

$$= \frac{f(\operatorname{crash rate} Y_s | \operatorname{class} = C) P(\operatorname{Class} C)}{\sum_{c=1}^{C} f(\operatorname{crash rate} Y_s | \operatorname{class} = C) P(\operatorname{Class} C)}$$
(9)

As mentioned previously, the application of the latent class modeling structure to the Tobit regression modeling framework in a safety context is a first.

MODEL ESTIMATION RESULTS

Random-Parameter Tobit Model

As shown in Tables 2 and 3, no parameters were found to be random for principal arterials and Interstates. However, two parameters were found to be random for major collectors and are shown in Table 4. To statistically assess the more significant log likelihood for major collectors—fixed- or random-parameter—the ensuing log likelihood ratio test was conducted.

$$\chi^2 = -2 \left[\log L(\beta_{\rm FP}) - \log L(\beta_{\rm RP}) \right] \tag{10}$$

where

- $logL(\beta_{FP}) = log likelihood at convergence for fixed-parameter model,$
- $logL(\beta_{RP}) = log likelihood at convergence for random-parameter model, and$
 - χ^2 = chi-square statistic with degrees of freedom equal to number of random parameters.

One more goodness-of-fit measure was applied, the Maddala pseudo- R^2 value (38):

$$R^{2} = 1 - e^{\left(\frac{-2[\log L(\beta) - \log L(0)]}{N}\right)}$$
(11)

Variable	Coefficient ^a	t-Stat. ^b	Partial Effect
Constant	0.12	4.48	
Speed limit (1 if 65 mph, 0 otherwise)	-0.11	-3.95	-4.09
Traffic control device (1 if no device, 0 otherwise)	-0.12	-4.33	-4.66
Road configuration (1 if 2-way and 2-way left-turn lane)	0.10	2.81	4.06
Heavy-vehicle AADT (1 if less than or equal to 300, 0 otherwise)	0.10	2.88	3.76
Passenger vehicle AADT (1 if greater than 10,500, 0 otherwise)	-0.22	-4.70	-8.44
Total AADT (1 if between 5,000 and 7,000, 0 otherwise)	-0.06	-1.71	-2.26

TABLE 2 Best-Fit Fixed-Parameter Tobit Regression Estimates for Principal Arterials

NOTE: Number of observations = 862; log likelihood at zero = -396.68; log likelihood at convergence = -355.12; $\chi^2 = 83.11$; Maddala pseudo- $R^2 = .092$. ^aSigma, $\sigma = 0.31$. ^bSigma, $\sigma = 26.85$.

TABLE 3 Bes	st-Fit Fixed-Parameter	Tobit Regression	Estimates for	 Interstates
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Variable	Coefficient ^a	t-Stat. ^b	Partial Effect
Constant	0.09	4.54	
Speed limit (1 if 75 mph, 0 otherwise)	-0.02	-2.20	-0.80
Road configuration (1 if 2-way and raised or depressed divider)	-0.03	-2.20	-1.37
Heavy-vehicle AADT (1 if between 2,000 and 3,000, 0 otherwise)	0.02	2.06	0.78
Horizontal geometrics (1 if curved, 0 otherwise)	0.02	2.36	0.95
Surface defects (1 if no surface defects, 0 otherwise)	-0.05	-2.81	-2.30
Passenger vehicle AADT (1 if greater than 15,000, 0 otherwise)	-0.03	-2.28	-1.30
Total AADT (1 when less than 6,500, 0 otherwise)	0.03	3.88	1.64

NOTE: Number of observations = 379; log likelihood at zero = 211.91; log likelihood at convergence = 242.54; $\chi^2 = 61.25$; Maddala pseudo- $R^2 = .149$. "Sigma, $\sigma = 0.06$.

 b Sigma, $\sigma = 20.74$.

TABLE 4 Best-Fit Random-Parameter Tobit Regression Estimates for Major Collectors

	Fixed-Parame	eter Tobit		Random-Parameter Tobit			
Variable	Coefficient	t-Stat.	Partial Effect	Coefficient	t-Stat.	Partial Effect	
Constant	-1.31	-5.94		-1.21	-5.40		
Speed limit (1 if less than or equal to 40 mph, 0 otherwise)	0.52	2.87	0.20	0.42	2.30	12.02	
Standard deviation of parameter, normally distributed	na	na	na	0.73	6.67	na	
Traffic control device (1 if stop sign, 0 otherwise)	0.45	2.22	0.18	0.40	2.01	11.59	
Horizontal geometrics (1 if straight, 0 otherwise)	0.91	4.38	0.36	0.73	3.67	20.96	
Road configuration (1 if 2-way and double-yellow painted divider, 0 otherwise)	0.70	3.10	0.27	0.36	1.53	10.41	
Standard deviation of parameter, normally distributed	na	na	na	0.99	5.45	na	
Sigma, σ	1.82	25.91		1.59	56.33		
Passenger vehicle AADT (1 if greater than 2,500, 0 otherwise)	-0.60	-2.90	-0.24	-0.50	-2.22	-14.31	
Total AADT (1 if less than 500, 0 otherwise)	1.03	5.61	0.41	0.97	4.88	27.90	

NOTE: Number of observations = 768 for fixed-parameter Tobit and random-parameter Tobit; log likelihood at zero = -1,003.14 for fixed-parameter Tobit and random-parameter Tobit; log likelihood at convergence = -967.07 for fixed-parameter Tobit and -926.07 for random-parameter Tobit; $\chi^2 = 72.13$ for fixed-parameter Tobit and 80.81 for random-parameter Tobit; Maddala pseudo- $R^2 = .090$ for fixed-parameter Tobit and .181 for random-parameter Tobit.

TABLE 5	Best-Fit Latent	Class Tobit	Regression	Estimates	for	Principal	Arterials
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	Latent Class 1		Latent Class 2		Latent Class 3		D
Variable	Coefficient	t-Stat.	Coefficient	t-Stat.	Coefficient	<i>t</i> -Stat.	Partial Effect
Constant	1.51	2.59	0.26	7.90	0.04	11.83	
Speed limit (1 if 65 mph, 0 otherwise)	0.05	0.06	-0.12	-3.25	0.00	-1.07	-0.45
Traffic control device (1 if no device, 0 otherwise)	-1.06	-2.06	-0.11	-3.15	-0.01	-1.93	-0.78
Road configuration (1 if 2-way and 2-way left-turn lane)	-0.94	-0.74	0.09	2.31	0.00	-0.12	0.04
Heavy-vehicle AADT (1 if less than or equal to 300, 0 otherwise)	0.93	1.86	0.05	1.34	0.00	0.69	0.49
Sigma, σ	0.61	2.31	0.15	10.52	0.02	15.46	
Class probability (t-statistic)	0.022 (2	.77)	0.262 (9	.06)	0.716 (25	5.24)	
Passenger vehicle AADT (1 if greater than 10,500, 0 otherwise)	-0.53	-0.10	-0.12	-1.58	-0.01	-1.56	-0.71
Total AADT (1 if between 5,000 and 7,000, 0 otherwise)	1.15	0.65	0.00	-0.05	0.00	-0.09	0.34

NOTE: Number of observations = 862; log likelihood at zero = 166.81; log likelihood at convergence = 195.60; Akaike information criterion = -339.20; Bayesian information criterion = -215.50.

where

 $logL(\beta) = log likelihood at convergence for best-fit model,$

logL(0) = log likelihood at zero, and

N = number of observations.

With regard to the principal arterial model, a chi-square statistic of 83.11 and 6 degrees of freedom indicated with 99.99% confidence that the fixed-parameter model is preferred over the model with simply the constant. For Interstates, a chi-square statistic of 61.25 and 6 degrees of freedom showed with 99.99% confidence that the fixed-parameter model is of more significance than the model with no estimated parameters. In the case of major collectors, where variables were found to be random, a chi-square statistic of 80.81 and 2 degrees of freedom demonstrated with 99.99% confidence that the random-parameter model is statistically preferred.

Latent Class Tobit Model

Latent class regression models for each road classification are shown in Tables 5 to 7. In line with previous studies, the number of latent classes for each model was selected by using the Bayesian information criterion (BIC); the number of latent classes that produced the smallest BIC was used (39, 40). However, Louviere et al. suggest that the smallest Akaike information criterion (AIC) be used to determine the best-fit number of classes, and this was the case for the major collector model (41). Yang found similar results with regard to the number of latent classes based on AIC (42).

The class split for each classification is highly significant and the best-fit number of classes is different for each model. Principal arterials have a best-fit model with three latent classes, major collectors with four latent classes, and Interstates with two latent classes.

DISCUSSION OF RESULTS

Tobit Model

High passenger vehicle AADT (PAADT) decreases crash rates for each road classification and has a significant impact on crash rates based on partial effects. "Partial effects" refers to a one-unit increase

TABLE 6 Be	st-Fit Latent	Class To	obit Regress	sion Estimates	for Ma	ior Collectors
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	Latent Class	1	Latent Class	2	Latent Class	3	Latent Class	4	
Variable	Coefficient	<i>t</i> -Stat.	Coefficient	<i>t</i> -Stat.	Coefficient	t-Stat.	Coefficient	t-Stat.	Partial Effect
Constant	2.11	0.71	0.08	3.83	0.07	0.93	0.43	1.69	
Speed limit (1 if less than or equal to 40 mph, 0 otherwise)	2.66	1.40	0.02	1.16	0.05	0.88	0.43	2.67	9.94
Traffic control device (1 if stop sign, 0 otherwise)	1.94	1.05	0.01	0.39	0.06	1.19	0.55	3.08	1.18
Horizontal geometrics (1 if straight, 0 otherwise)	-1.28	-0.61	0.02	1.00	0.13	2.17	0.02	0.07	-5.24
Road configuration (2-way and double-yellow painted divider, 0 otherwise)	2.46	1.07	0.02	0.81	0.13	1.85	-0.09	-0.44	6.56
Passenger vehicle AADT (1 if greater than 2,500, 0 otherwise)	-3.71	-1.04	-0.03	-1.39	-0.06	-0.90	0.35	1.87	-5.91
Sigma, σ	3.10	3.83	0.07	9.64	0.16	5.44	0.44	5.38	
Class probability (t-statistic)	0.076 (3	.93)	0.577 (13	3.22)	0.227 (4	.97)	0.120 (4	.98)	
Total AADT (1 if less than 500, 0 otherwise)	2.28	1.17	0.01	0.33	0.29	5.52	0.74	4.26	7.00

NOTE: Number of observations = 768; log likelihood at zero = -437.00; log likelihood at convergence = -414.63; Akaike information criterion = 899.30; Bayesian information criterion = 1,061.80.

TABLE 7	Best-Fit	Latent Class	Tobit F	Regression	Estimates	for	Interstates
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	Latent Class	1	Latent Class 2	2		
Variable	Coefficient <i>t</i> -Stat		Coefficient	t-Stat.	Partial Effect	
Constant	0.72	5.05	-0.01	-0.20		
Speed limit (1 if 75 mph, 0 otherwise)	0.00	0.02	-0.01	-1.49	-0.37	
Road configuration (1 if 2-way and raised or depressed divider)	-0.01	-0.24	0.01	0.48	0.23	
Heavy-vehicle AADT (1 if between 2,000 and 3,000, 0 otherwise)	0.00	-0.12	0.02	2.32	0.65	
Horizontal geometrics (1 if curved, 0 otherwise)	-0.01	-0.55	0.02	1.63	0.24	
Surface defects (1 if no surface defects, 0 otherwise)	-0.70	-4.81	0.01	0.36	-10.78	
Sigma, σ	0.03	4.21	0.04	11.84		
Passenger vehicle AADT (1 if greater than 15,000, 0 otherwise)	-0.01	-0.29	-0.02	-0.61	-0.77	
Class probability (<i>t</i> -statistic)	0.361 (2	.30)	0.639 (4	.06)		
Total AADT (1 when less than 6,500, 0 otherwise)	0.00	-0.10	0.03	2.23	0.69	

NOTE: Number of observations = 379; log likelihood at zero = 363.56; log likelihood at convergence = 385.48; Akaike information criterion = -733.00; Bayesian information criterion = -658.10.

in an exposure variable while all others are held constant and the outcome it has on heavy-vehicle crash rates. Partial effects show that PAADT greater than 10,500 on principal arterials decreases the number of heavy-vehicle crashes per MVMT by 0.084. Similarly, PAADT greater than 2,500 on major collectors reduces the number of heavy-vehicle crashes per MVMT by 0.143 and PAADT greater than 15,000 on Interstates reduces heavy-vehicle crashes by 0.013 per MVMT. Conversely, low total AADT (passenger vehicles and heavy vehicles) increases crash rates. For instance, partial effects indicate that AADT less than 500 on major collectors increases the number of heavy-vehicle crashes by 0.279 per MVMT. Likewise, AADT less than 6,500 on Interstates results in an increase of 0.016 crashes per MVMT. These findings are in line with previous work (21, 22, 30, 43) in which lower AADT increases crash rates while higher AADT decreases crash rates. The same literature finds that the presence of heavy-vehicle traffic decreases crash rates, yet the current study found that the presence of heavy-vehicle traffic increased crash rates for principal arterials and Interstates. A possible explanation could be that such a finding is exclusive to the state of Idaho.

Two parameters were found to be random on major collectors on the basis of the statistical significance of the mean and standard deviation. The estimated parameter for a speed limit less than or equal to 40 mph was found to be random and normally distributed with a mean of 0.42 and standard deviation of 0.73. This finding suggests that for 28.2% of heavy vehicles the estimated parameter mean is less than zero and for 71.8% it is greater than zero. In other words, lower speed limits on major collectors decrease crash rates for 28.2% of heavy vehicles and increase crash rates for 71.8%. Chen et al., however, found that lower speed limits increase crash rates for all observations when the random-effects Tobit model is used; this finding possibly indicates that the random-effects approach does not account for all the heterogeneity in their data set (24). In contrast, high speed limits decrease crash rates on principal arterials and Interstates. Speed limits of 65 mph decrease crash rates on principal arterials and partial effects show a reduction of 0.041 crash per MVMT. Interstates with a speed limit equal to 75 mph see a decrease in crash rates, and partial effects indicate a decrease of 0.008 heavy-vehicle crash per MVMT. Although higher speed limits are prone to cause more severe crashes, they have been shown to reduce crash rates [see work by Dutta and Noyce

(44) for a thorough literature review regarding implications of high speed limits].

As for road configuration, two-way major collectors with a double yellow-painted divider were found to have a normally distributed random parameter. With a mean of 0.36 and standard deviation of 0.99, this configuration decreases crash rates for 35.8% of heavy vehicles and increases crashes rates for 64.2%. Two-way Interstates with raised or depressed dividers experience a reduction in crash rates and have a partial effect of -0.014. Road configuration, however, on principal arterials increases crash rates; partial effects suggest that two-way principal arterials with a two-way left-turn lane result in an increase of 0.041 heavy-vehicle crashes per MVMT.

With regard to horizontal geometrics, straight and curved conditions increase crash rates for major collectors and Interstates. Major collectors experience an increase in crash rates because of straight horizontal geometrics, and partial effects show an increase of 0.210 heavy-vehicle crash per MVMT. Horizontal curves increase crash rates on Interstates, though there is just a 0.010 increase. Curved geometrics were found to increase crash risk by Yu et al. (29), whereas the degree of curvature was found to increase crash rates by Chen et al. (24).

Other notable factors contributing to the crash rate are traffic control devices and surface defects. No traffic control devices on principal arterials decrease crash rates, and partial effects indicate a reduction of 0.047 heavy-vehicle crashes per MVMT. In contrast, stop signs on major collectors increase heavy-vehicle crashes by 0.116 crash per MVMT. Interstates with no surface defects decrease crash rates. This variable has the largest effect on Interstate crash rates, since partial effects suggest a decrease of 0.023.

Latent Class Tobit Model

The presence of latent classes suggests that various explanatory variables are heterogeneous. For example, a two-way road with a two-way left-turn lane on a principal arterial is positively significant in latent Class 2, but it is negative and not significant in latent Classes 1 and 3. These results indicate the presence of heterogeneity and negative and positive impacts on crash rates with such road configurations (45). Similar findings are presented in each latent class specification and exist for each variable.

With regard to class probability, the prior probabilities for principal arterials indicate that the probability that a crash belongs to latent Class 3 is the highest at 0.716. This finding is seen in the posterior probabilities, since 80.7% of the heavy-vehicle crashes belong to latent Class 3, whereas 17.9% and 1.4% belong to latent Classes 1 and 2, respectively. For major collectors, there is a 0.577 prior probability that crashes belong to latent Class 2. Posterior probabilities suggest that this is so, since 72.4% belong to latent Classes 1, 3 and 4, respectively. Prior class probabilities for Interstates indicate a 0.361 probability that a heavy-vehicle crash belongs to latent Class 1 and a 0.639 probability that it belongs to latent Class 2. Posterior probabilities agree: 10.8% of heavy-vehicle crashes belong to latent Class 1 and 89.2% belong to latent Class 2.

For principal arterials, the partial effects of the Tobit model are significantly greater than those of the latent class model. The partial effect for PAADT greater than 10,500 using the Tobit model was -0.084, but according to the latent class model, this PAADT decreases the number of heavy-vehicle crashes by 0.007 per MVMT. Overall, the partial effects for the latent class model were much less than those for the Tobit model.

For major collectors, latent class partial effects were substantially less when compared with the Tobit model. For example, PAADT greater than 2,500 has a partial effect of -0.143 for the Tobit model, yet the same variable based on the latent class model results in a reduction of 0.059 heavy-vehicle crashes per MVMT.

Interstates, however, experienced a decrease in partial effects for some variables and an increase in others, even a change in signs for one variable. For instance, two-way Interstates with a raised or depressed divider has a partial effect of -0.014 for the Tobit model, and the latent class model has the opposite effect and results in an increase of 0.002 heavy-vehicle crashes per MVMT. The partial effect of the Tobit model for PAADT greater than 15,000 is -0.013but increases to -0.008 for the latent class model. Interstates with no surface defects decrease the number of heavy-vehicle crashes per MVMT by 0.023 according to the Tobit model and increase the reduction to 0.108 according to the latent class estimations.

Model Comparison

To determine the best-fit model for the Idaho crash data, three metrics were evaluated: overall model fit, partial effect inferences, and the rate of prediction of actual crash rate values. To illustrate, the latent class approach for each road classification had a better overall model fit. Log likelihoods are typically negative; however, it is possible to see positive values for regression of a continuous dependent variable. In such a case, the greater the value is (if positive), the better the fit of the model is. In terms of partial effects, the latent class framework identified different high-impact variables and partial effects were much less, as were the partial effects for the random-parameter model. The fit of the actual crash rates versus the predicted crash rates for both regression estimates is as follows:

Tobit R^2	Latent Class R ²
.09	.76
.29	.88
.22	.72
	<i>Tobit R</i> ² .09 .29 .22

The corresponding plots are presented in Figure 2. The plots visually illustrate that the Tobit model substantially underpredicted the crash rates for each road classification and that the latent class model outperformed the Tobit model significantly.

SUMMARY AND INSIGHTS

This study utilized two specific econometric frameworks, namely, random-parameter Tobit regression and latent class Tobit regression, to determine factors that contribute to the number of heavy-vehicle crashes per MVMT by roadway classification while identifying a preferred method to account for unobserved heterogeneity. Policereported crash data are often missing key variables (e.g., the variables are not on data collection forms) and vary across existing variables; therefore, utilizing the random-parameter Tobit method allows the analyst to account for heterogeneity by defining a distribution. The latent class approach also accounts for heterogeneity, but no distribution is defined and the parameters are permitted to vary across a specified number of classes. Using goodness-of-fit measures and the rate of prediction, the estimates of the two approaches were examined.

The Idaho case study provides new insights into crash rates by roadway classification. Different road configurations, horizontal geometrics, and traffic control devices were found to be significant for each road classification. A specific road configuration was found to decrease crash rates for major collectors and Interstates but to increase crash rates on principal arterials. Curved horizontal geometrics increase crash rates on Interstates and straight horizontal geometrics increase crash rates on major collectors. Stop signs on major collectors increase crash rates, yet no traffic control devices on principal arterials decrease crash rates. High speed limits decrease crash crates on principal arterials and Interstates, and lower speed limits increase crash rates for the majority of heavy vehicles on major collectors. The most common insight from this study is that high traffic volumes decrease crash rates and low volumes increase crash rates. With that in mind, unlike previous work, this study found that the presence of heavy vehicles has the potential to increase crash rates.

To assess the accuracy of the two frameworks, the actual crash rates and predicted crash rates were plotted and the Pearson product moment correlation coefficient was provided for each. The latent class approach outperformed the traditional Tobit method for each road classification and, as a result, should be considered in future crash rate analyses. In addition, the sample size may indicate what information criterion (AIC or BIC) to use when the correct number of latent classes is selected. However, results indicating a better fit for the latent class approach are entirely data-specific. Although the current study has found that the latent class model better describes the Idaho crash data, there is the potential that the latent class model may not be better suited in crash data sets of other states. This finding strongly suggests that further work be conducted in the comparison of these two heterogeneity methods. Unfortunately, this is an inherent limitation of the latent class modeling framework. Although the model, in this case, captures more heterogeneity and provides a better fit for the data, this result is not always the case.

In summary, this study exhibited two distinct methodologies to model crash rates while accounting for heterogeneity. Factors that contribute to crash rates differ depending on road classification and in future work they should be analyzed separately. Such findings can assist with safety measures in Idaho by providing transportation agencies, engineers, planners, and policy makers with contributing crash rate factors coupled with more precision. For example, road



FIGURE 2 Actual crash rates versus predicted crash rates: (a) latent class model, principal arterials; (b) Tobit model, principal arterials; (c) latent class model, major collectors; (d) Tobit model, major collectors; (e) latent class model, interstates; and (f) Tobit model, interstates.

configuration was found to affect crash rates by road classification, and restriping to reconfigure configurations can be an economically viable solution to reduce heavy-vehicle crash rates in Idaho. In addition, stop signs were found to increase crash rates on major collectors and a possible explanation could be ineffective stop sign location; hence, relocating stop signs is yet another economically viable solution to reduce heavy-vehicle crash rates in Idaho. The presented framework—censored latent class regression—should strongly be considered when future crash rate analyses are conducted, as well as analysis by roadway classification in other geographic regions.

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