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A Two-Stage Procedure for Determining Unsaturated Hydraulic Characteristics using a Syringe Pump and Outflow Observations

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ABSTRACT

A fast two-stage methodology for determining unsaturated flow characteristics is presented. The procedure builds on direct measurement of the retention characteristic using a syringe pump technique, combined with inverse estimation of the hydraulic conductivity characteristic based on one-step outflow experiments. The direct measurements are obtained with a commercial syringe pump, which continuously withdraws fluid from a soil sample at a very low and accurate flow rate, thus providing the water content in the soil sample. The retention curve is then established by simultaneously monitoring the capillary pressure. The measured retention curves were compared with those obtained by static methods and found to agree closely. Subsequently, a transient outflow experiment is carried out on the same soil sample, in the same apparatus, and the information is obtained on a time-scale of days. The one-step outflow data and the independently measured retention data are included in the objective function of a traditional least-squares minimization routine, providing unique estimates of the unsaturated hydraulic characteristics by means of numerical inversion of Richards equation. As opposed to what is often assumed in practical applications, we found it necessary to allow the exponent (γ) of van Genuchten's hydraulic conductivity expression to vary to obtain satisfactory estimates. Optimized hydraulic conductivity estimates were ultimately compared with directly measured values, and visual agreement was quite satisfactory. However, we encountered numerical instabilities of the optimization code used (SFIT), a flaw that may constitute a potential problem when using this code for the optimization.

FLOW THROUGH UNSATURATED POROUS MEDIA is generally described by the highly nonlinear Richards (1931) equation. Prediction of unsaturated flow by solving the Richards equation requires knowledge of the soil hydraulic properties [retention function θ(ψ) and hydraulic conductivity function K(ψ)]. Traditionally, these relationships have been estimated independently from steady state experiments (Dirksen, 1990; Klute, 1986; Richards, 1967; Corey, 1985) that are time-consuming and elaborate, because several stages of steady state conditions have to be reached. Thus, in recent years, research effort has been focusing on developing faster alternative methods for determining the unsaturated hydraulic properties, for instance transient outflow methods.

The outflow method was first introduced by Gardner (1956), who used analytical techniques to obtain the unsaturated hydraulic conductivity and diffusivity from transient measurements. This technique has been used in several studies since then (Gardner, 1956; Gupta et al., 1974; Zachmann et al., 1981, 1982; Hornung, 1983; Valiantzas et al., 1988; Valiantzas and Kerkides, 1990). However, it was through the work of Kool et al. (1985), Kool and Parker (1985), and Parker et al. (1985), who introduced the so-called one-step approach, that the method became more versatile and gained greater acceptance. In their procedure, cumulative outflow from an initially saturated soil sample as induced by a step increase in applied pneumatic pressure is simulated by numerical inversion of the nonlinear Richards equation based on parameterized hydraulic functions. Parameters in the expressions for both retention and hydraulic conductivity relations are adjusted until simulations match measurements. The numerical approach provides flexibility with respect to initial and boundary conditions. The one-step method has been adopted in several investigations since Kool and Parker reported their work in 1985. However, many researchers have encountered problems with nonuniqueness and instability in the optimization process.

Several researchers have tried to solve the optimization problems by including more information about the flow system, for instance by measuring independently the capillary pressure in the sample during outflow (Eching and Hopmans, 1993), considering multi-step changes in boundary conditions (van Dam et al., 1994; Eching et al., 1994), combining these two approaches (Eching et al., 1994), or including weighted, independently measured, retention data in the objective function (van Dam et al., 1992). Most of these approaches have been quite successful in providing more unique estimates of the hydraulic parameters, in some cases, however, at the cost of experimental simplicity. Toorman et al. (1992) corroborated theoretically the value of experimental refinements by looking at response surfaces for the optimization. They found that including additional information, for instance capillary pressure data, in the optimization greatly improves the parameter estimation. From the research conducted so far it appears that auxiliary information must be provided to obtain reliable estimates with the commonly ill-posed one-step methodology.

A convenient approach to improving the optimization is to include independently measured θ(ψ) data in the objective function. The traditional methods for measuring θ(ψ) data [hanging column method, pressure cell method, etc. (Haines, 1930; Richards, 1941; Klute, 1986)] are generally very time-consuming and elaborate. Along with the problems of their inverse estimation, the experimental tediousness of measuring static retention characteristics has prompted the development of faster, dynamic procedures for direct measurement of the retention characteristic. Previously proposed dynamic approaches (Klute, 1986) have been confined to the measurement range of

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a tensiometer, thus imposing certain limitations on the method. Klute (1986) reports that the drainage data obtained by the dynamic method often implies higher retention than the data from the conventional static method; however, no explanation is offered for this discrepancy. Dirksen (1979) used a syringe pump for flux-controlled sorptivity measurements with the objective of estimating unsaturated hydraulic conductivities from the measured diffusivities. This implies that the wetting retention characteristic is also determined. Dirksen (1979) found that the flux boundary condition provided more accurate experimental control than the previously used potential-controlled boundary condition (Dirksen, 1975). Salehzadeh and Demond (1994) presented a new, relatively complex apparatus for rapid automated measurement of unsaturated soil properties. According to the authors, the apparatus can be used for measuring both retention and hydraulic conductivity functions, but the estimation of the hydraulic conductivity was not convincing and the ability of the apparatus to produce accurate capillary pressure measurements was, therefore, considered more important by the authors. Clearly, a need for a fast and accurate procedure for obtaining retention curves still exists, particularly a simple method that does not involve a complicated setup of flow and pressure regulators, valves, and bleeders. The syringe pump method, first used for retention curve measurements by Znidarcic et al. (1991) and Manna (1991) provides such a simple and reliable method. However, Manna (1991) only tested the technique on a single soil type and for a small range of suctions.

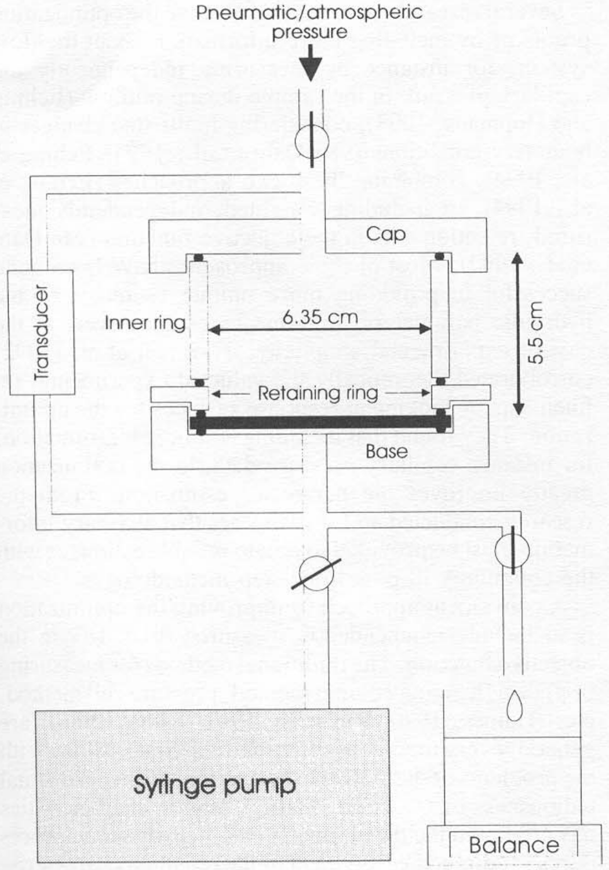

Fig. 1. Laboratory setup with pressure cell, syringe pump, differential transducer, and electronic balance.

In this study, we present a relative fast and convenient methodology for determining both retention and hydraulic conductivity characteristics on a single soil sample. The objective was (i) to improve the syringe pump technique for measuring retention characteristics, to test it on a range of soil types, compare the results with those obtained by other methods, and to analyze the validity of its assumptions by numerical simulation, and subsequently, the goal was (ii) to combine the independently measured retention data with transient outflow data, obtained from a one-step experiment carried out on the same re-saturated soil sample. By this procedure, it is possible to inversely estimate the hydraulic conductivity function, thus obtaining the complete set of parameters describing unsaturated flow within a time-frame of days for the mostly sandy materials considered in this study. We used different estimation schemes, including an approach where a certain degree of decoupling of retention and hydraulic conductivity expressions was allowed to add more flexibility to the optimization, based on the idea that the two functions are not necessarily explicitly related. The resulting parameter estimates were ultimately compared with directly measured data.

METHODS

Materials

Six different sands ranging from coarse to fine grain sizes were used for verifying our combined two-stage methodology. The grade no. 30, 70, and 125 sands [following the nomenclature of U.S. standard sieve sizes (0.600-, 0.212-, and 0.120-mm sieve sizes, respectively)] were highly uniform, crushed silica (Granusil) and were categorized as coarse, medium, and fine sands, respectively. The three additional sands were natural silica sands no. 00, 0, and 1 representing fine, fine–medium, and coarse sands. All the sands used are commercially available sorted sands and thus have a relatively narrow pore-size distribution. In addition to the sands, a consolidated porous material derived from vermiculite clay was used for testing the syringe pump technique on a finer, less permeable material.

Syringe Pump Experimental Procedure

The syringe pump laboratory set-up is shown schematically in Fig. 1. The main components are a pressure cell (6.35 cm diameter and 5.5 cm high), a syringe pump, and a pressure transducer. The cell is made of acrylic glass and rubber o-rings to seal mating surfaces. The retaining ring clamps a porous plate to the base, with o-rings sealing each joint. The porous plates used are a 7.1-mm-thick, ½-bar (or 1-bar for the vermiculite material) ceramic plate, both from Soil Moisture Equipment Corp. (Santa Barbara, CA) or a 4-mm-thick sintered bronze plate with a bubble pressure of ≈ 140 cm from Dansk Sintermetal A/S (Haderslev, Denmark). Saturated hydraulic conductivities are $4.2\ 10^{-5}$ m s^{-1} and $8.3\ 10^{-8}$ m s^{-1} for the bronze and ceramic plates, respectively. The outer 7 mm of the plates and the sides are either epoxy-coated (ceramic plates) or tin-soldered (bronze plate) to form an impervious surface that seals against the o-rings. The water phase is continuous from below the porous plate into the transducer and the syringe pump. The pumps used are either a Model 906 mechanical syringe pump or a Model 44 programmable electronic syringe pump, both from Harvard Apparatus, Inc. (Woburn, MA). The piston velocity can be varied in the range from $2.12\ 10^{-8}$ m s^{-1} to $1.06\ 10^{-3}$ m s^{-1} (Model 906) or from $3.0\ 10^{-9}$ m s^{-1} to $3.2\ 10^{-3}$ m s^{-1} (Model 44). With different piston sizes, the flow rates can thus be varied across large ranges. Operating successively in withdrawal and infusion modes, the pumps can be used to obtain both drainage and imbibition curves.

The transducer is connected to the water phase underneath the porous plate, thus measuring the difference between atmospheric pressure and the water phase pressure, i.e., the capillary pressure in the soil sample, immediately above the porous plate. It is assumed that the pressure in the water underneath the porous plate represents the capillary pressure in the soil sample immediately above the porous plate, i.e., that the pressure gradient across the porous plate is negligible. Measuring the capillary pressure in the water phase underneath the porous plate is superior to using inserted tensiometers, because tensiometers often lose hydraulic contact with the soil at relatively low water contents. This is particularly a problem when relatively coarse materials are considered. Furthermore, insertion of a tensiometer may cause disturbance in the sample.

The procedure for measuring a drainage curve is as follows. The presaturated porous plate is carefully placed in the water-filled base of the pressure cell, avoiding entrapment of air bubbles. The retaining ring is placed on top and the porous plate clamped between the base and the retaining ring. Excess water is removed and the inner ring placed on the retaining ring. To avoid saturation of the sand during packing, the valve underneath the cell is then closed, and the cell is packed with sand and the porosity subsequently determined from the amount of sand used. During packing, the sand is compacted twice at one-third and two-third levels using a lead weight. The cap is placed on top and the sample saturated from below with distilled, de-aired water. When the water level reaches the top of the sample, a pressure of ≈ 10 m is applied to the water phase for ≈ 15 h to dissolve entrapped air bubbles. When the water phase pressure is released, the sample is flushed with approximately two pore volumes of fresh de-aired water at a low flow rate (to avoid disturbance of the sample) and subsequently equilibrated with a reservoir such that the pressure is zero at the top of the sample. The syringe pump is started in withdrawal mode and the drainage curve determined from corresponding values of water content and capillary pressure. The water content is derived from the volume withdrawn by the syringe pump. If an imbibition curve is required, the pump is switched to infusion mode after the drainage curve has been obtained. Experimental control is computerized, requiring no physical intervention between drainage and imbibition stages. In this study, we used flow rates between 4.2×10^{-4} mL s^{-1} and 5.6×10^{-6} mL s^{-1}, and each curve could then be determined in ≈ 3 d.

The syringe pump technique was tested on eight different sand types ranging from coarse to fine sands and on the consolidated porous material derived from vermiculite clay. However, for brevity, we have chosen to only present the results of the tests on one of the sand types (results were typical for the remaining sand types) and on the consolidated clay material. The sand (no. 0) was subjected to one static and four dynamic tests on replicate samples to compare the syringe pump method to traditional methods and to test for reproducibility, while one static and two dynamic test were performed on the vermiculite clay.

The dynamic tests on sand no. 0 and the vermiculite clay soil were carried out using a combination of high and low flow rates. It was found during the first experiments that using lower flow rates near full and residual saturation resulted in more stable readings and thus more accurate retention curves. In addition, it was found that it is possible to use a higher flow rate than estimated from numerical simulations (discussed below) in the intermediate stage between full and residual saturation and thereby minimize the experimental time demand. Multiple flow rates were thus used in the remaining dynamic experiments. Generally, the low flow rate used was half the rate estimated from the numerical simulations and the high flow rate used in the intermediate stage was one-and-a-half the estimated rate.

The static measurements of the retention characteristics for the sands were carried out in the syringe pump laboratory setup, but the pump was used discontinuously to withdraw given volumes of water from the sample at the same rate as used in the dynamic experiments. After the water was withdrawn, the pump was stopped, hydrostatic equilibrium was awaited (considered to be secured when the change in capillary pressure was <2 mm within an hour), and a set of corresponding values of pressure and water content was hereby obtained. By repeating this step-wise procedure, consecutive points on the static retention curve were established. In the case of the consolidated clay material, a traditional pressure cell procedure was used for obtaining the static measurements.

The proposed technique is not applicable close to residual saturation where the water phase no longer forms a continuous phase, i.e., there is no liquid flow at lower water contents. In this regime, however, the water flow is no longer of practical significance for most applications. If retention characteristics at higher suctions are of interest (for instance when evapotranspiration is considered), the proposed method has to be combined with a traditional pressure cell technique. The technique is only valid when the basic assumptions apply. We carried out a number of numerical simulations to verify that this is generally the case and that the syringe pump technique provides results consistent with those obtained by traditional techniques.

Syringe Pump Technique – Numerical Simulations

Of particular concern are the questions whether (i) the pressure distribution in the soil sample is close to hydrostatic during the experiment as is the case when using traditional equilibrium techniques and (ii) the pressure in the water underneath the porous plate actually represents the capillary pressure in the soil sample above the porous plate.

The model used for the numerical simulations was a Galerkin type linear finite element solution of the Richards equation [SIM2D, Gregersen (1995)]. The simulations were performed on no. 70 sand, and the van Genuchten parameters (van Genuchten, 1980) used in the model were obtained from the retention characteristic already measured using the syringe pump method. The porous plate conductivity used was that of the ceramic plate, i.e., $K_p = 8.3\ 10^{-8}$ m s^{-1}. Simulations were performed for the flow rate used in the physical experiments (2.5×10^{-4} mL s^{-1} or 7.9×10^{-8} m s^{-1}), as well as 0.1, 10, and 100 times this flow rate. Note that, for simplicity, a constant flow rate was used throughout the experiment in these verifying simulations. This procedure shows that finding a permissible flow rate for the syringe pump is potentially an iterative process.

One-Step Experimental Procedure

The one-step experiments were carried out in the same laboratory setup (Fig. 1) as used for measuring the retention characteristics. After re-saturation and equilibration at a suction of 1 to 2 mm at the bottom of the sample [for the reasons mentioned in Hopmans et al. (1992) and Corey (1992)], a pneumatic pressure was imposed on the air phase inducing unsaturated flow in the soil sample with the porous plate remaining saturated. Pneumatic pressures of ≈500 and 110 cm were used for the low-flow and high-flow plates, respectively. The outlet at the bottom of the pressure cell was placed over a computer-monitored balance for measuring cumulative outflow. Using the balance for measuring the cumulative outflow eliminates the problem of an ill-defined boundary condition present when a burette is used for measuring the outflow. In such a setup, the water level of the burette moves upward as the outflowing water accumulates and therefore constitutes a continuously changing boundary condition.

Long-Column Experimental Procedure

The long-column technique (Corey, 1985) was used to directly measure the unsaturated hydraulic conductivity of the six sands. The laboratory setup consists of a long (140 cm) acrylic glass column with tensiometers located at two different levels and a water table at the bottom. The experiments were carried out under the assumption that gravity flow prevails at some distance from the water table (Corey, 1985), and thus, the capillary suction is uniform in this domain. This implies that the unsaturated hydraulic conductivity, at the measured capillary pressure, equals the applied input flow rate. By changing the input flow rate incrementally, waiting for stationarity to develop, and measuring the corresponding pressure, consecutive points on the unsaturated hydraulic conductivity curve could be obtained. The tensiometers were placed at two different levels, and the pressure was monitored continuously to ensure that the assumption of a unit gradient was actually valid.

Parameter Estimation

Simulation of the one-step experiment was performed by solving Richards equation subject to the initial and boundary conditions depicted by Kool and Parker (1987). To apply this inverse approach, the soil hydraulic functions must be described by parametric models with a limited number of parameters. Russo (1988) investigated the van Genuchten (1980), Brooks and Corey (1964), and Gardner-Russo (Russo, 1988) soil hydraulic models and concluded that the van Genuchten model was superior in the one-step parameter estimation procedure. Subsequently, though, Russo et al. (1991) pointed out that the larger number of parameters in the van Genuchten model may enhance the problems of nonuniqueness and parameter correlation often encountered in the inverse solution. Nevertheless, in this study, the retention curve was parameterized by the model of van Genuchten (1980), given as

$$S_e = \frac{1}{(1 + (\alpha\psi)^n)^{1-1/n}} \qquad \psi > 0$$

where S_e is effective saturation, $S_e = (\theta - \theta_r)/(\theta_s - \theta_r)$, θ is the water content, θ_s and θ_r are the water contents at full and residual saturation, respectively, ψ is matric suction, and α and n are empirical parameters. Combining this expression with Mualem's (1976) relation for the unsaturated hydraulic conductivity results in

$$K = K_s S_e^{\gamma}[1 - (1 - S_e^{1/m})^m]^2$$

where K_s is the saturated hydraulic conductivity and $m = 1 - 1/n$. The fitting parameter, γ, is commonly fixed at a value of 0.5 so that the relationship between retention and hydraulic conductivity is unambiguous (i.e., identical parameters as implied by the original derivation of van Genuchten, 1980). In this study, we chose to optimize this parameter as well, to allow a certain degree of decoupling of retention and hydraulic conductivity and thereby add another degree of freedom to the optimization.

The program SFIT by Kool and Parker (1987) was used for estimating the parameters in the hydraulic functions. The SFIT program uses a nonlinear weighted least-squares minimization routine based on the Levenberg-Marquardt method for the optimization. The direct problem, i.e., the numerical solution of the Richards equation, is solved by a finite element routine developed by van Genuchten (1982, 1987). If retention data is included in the optimization, the following objective function $E(\mathbf{b})$ is minimized:

$$E(\mathbf{b}) = \sum_{i=1}^{N} [w_i \{Q(t_i) - Q_o(\mathbf{b},t_i)\}]^2 + \sum_{j=1}^{M} [v_j W \{\theta(\psi_j) - \theta_o(\mathbf{b},\psi_j)\}]^2$$

where $Q(t_i)$ is the cumulative outflow measured at time t_i, $Q_o(\mathbf{b},t_i)$ is the optimized cumulative outflow corresponding to the trial parameter vector $\mathbf{b} = (\alpha, n, \theta_r, \theta_s, K_s, \gamma)$, $\theta(\psi_j)$ is the measured retention data, $\theta_o(\mathbf{b},\psi_j)$ represents the water content for the trial parameter vector $\mathbf{b}$, and w_i and v_j are weight factors accounting for differences in measurement units. W is a normalizing factor of the retention data:

$$W = \frac{1}{N}\sum_{i=1}^{N} Q(t_i) \Big/ \frac{1}{M}\sum_{j=1}^{M} \theta(\psi_j)$$

The optimization problem is solved by adjusting the parameters in the trial vector **b** until the weighted sum of squares is minimized.

The parameters K_s, θ_s, and θ_r were fixed at their measured values in the optimization since these properties can be measured directly in the laboratory at reasonable accuracy. Three different estimation procedures were attempted: (i) estimation of α and n directly from retention data, using the nonlinear least-squares optimization program, RETC (Yates et al., 1992) and assuming $\gamma = 0.5$ in the prediction of the unsaturated hydraulic conductivity relation. This approach was attempted because it is used extensively in investigations where only retention characteristics are available, for instance in a majority of field investigations; (ii) inverse estimation of α and n including data from both one-step outflow experiments and retention measurements in the objective function of SFIT, with γ fixed at 0.5; (iii) analogous to (ii) but allowing γ to vary to provide another degree of freedom to the optimization. This implies a partially decoupled inverse estimation of α, n, and γ from one-step outflow experiments and retention data in the sense that we relax the parallel stream tube conceptual model, which implies identical values of α and n throughout by forcing $\gamma = 0.5$.

RESULTS AND DISCUSSION

Numerical Simulations

Distributions of pressure and water content according to the numerical simulations at the three highest of the four previously mentioned flow rates (1, 10, and 100 times the used rate) are shown in Fig. 2 for the no. 70 sand. The figures show pressure and water content as a function of level above the bottom of the cell; i.e.,

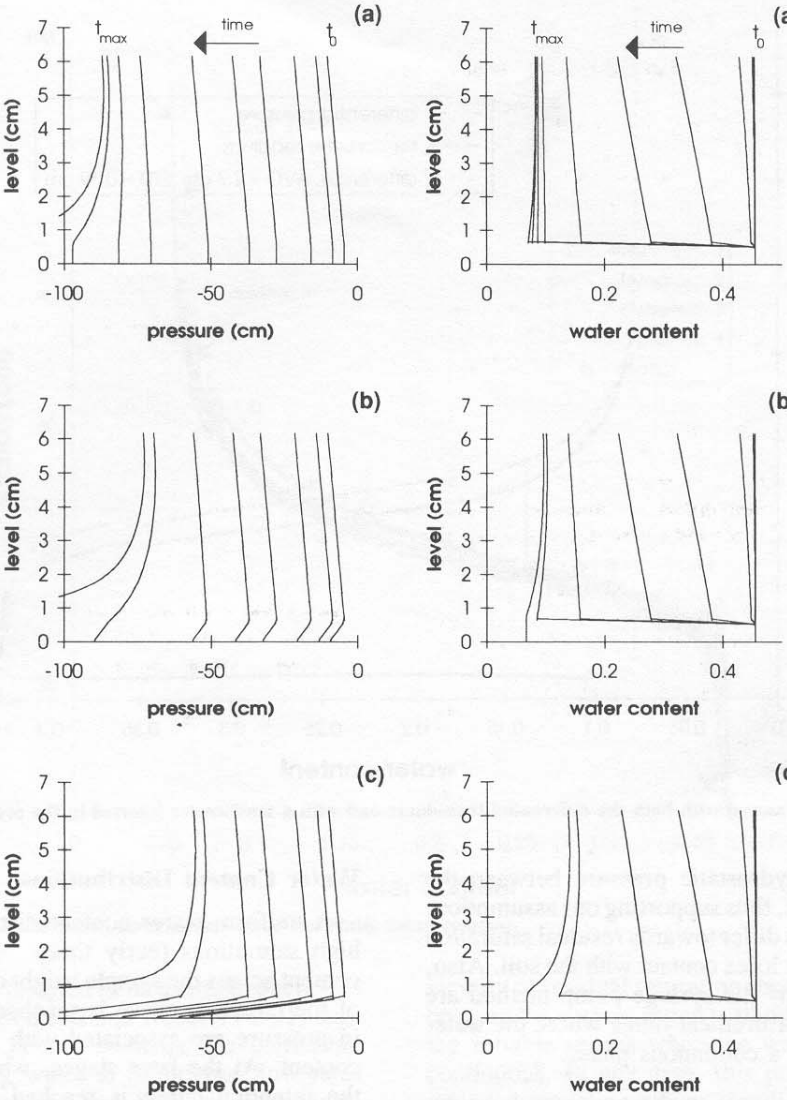

Fig. 2. Numerical simulations of pressure and water content distributions for the no. 70 sand at different times for three multiples of the experimental flow rate; (a) one time, (b) ten times, (c) 100 times. For corresponding curves the same amount of water has been withdrawn from the sample (t = time).

the bottommost 0.71 cm represent the conditions in the porous plate. The curves shown in the figure illustrate the conditions in the sample at different times during the outflow process (time increasing from right to left).

Pressure Distributions

Simulations considering the flow rate used in the physical experiments (top row in Fig. 2) show that the pressure distributions in the sample are rather close to hydrostatic (the simulated gradient was linear and deviated by only 2.5% from unity), while a nearly uniform pressure distribution is present in the low-flow ceramic porous plate because of its higher resistance to flow. (The higher resistance to flow in the porous plate causes a pressure drop over the plate which is dependent on the flow rate; see also the middle and bottom rows of Fig. 2). However, when the water content of the sample approaches residual ($t \rightarrow t_{max}$), the pressure distribution deviates from hydrostatic conditions. Therefore, the pressure recorded by the transducer is no longer representative of the conditions in the sample, and one of the basic assumptions of the method is violated. The deviation from a hydrostatic pressure distribution becomes larger at higher flow rates (middle and bottom rows in Fig. 2). Additionally, the impact of the porous plate becomes particularly critical with increasing flow rate, as a considerable pressure gradient develops due to plate impedance such that the pressure registered underneath the plate is not representative of the actual pressure in the sample. The question of representativeness of the pressure measurements, when using our estimated flow rate, was also addressed in some of the physical experiments. Tensiometer readings made with a tensiometer installed in the soil sample measuring the capillary pressure directly in the soil were compared with the measurements made in the water underneath the porous plate (Fig. 3). They varied only

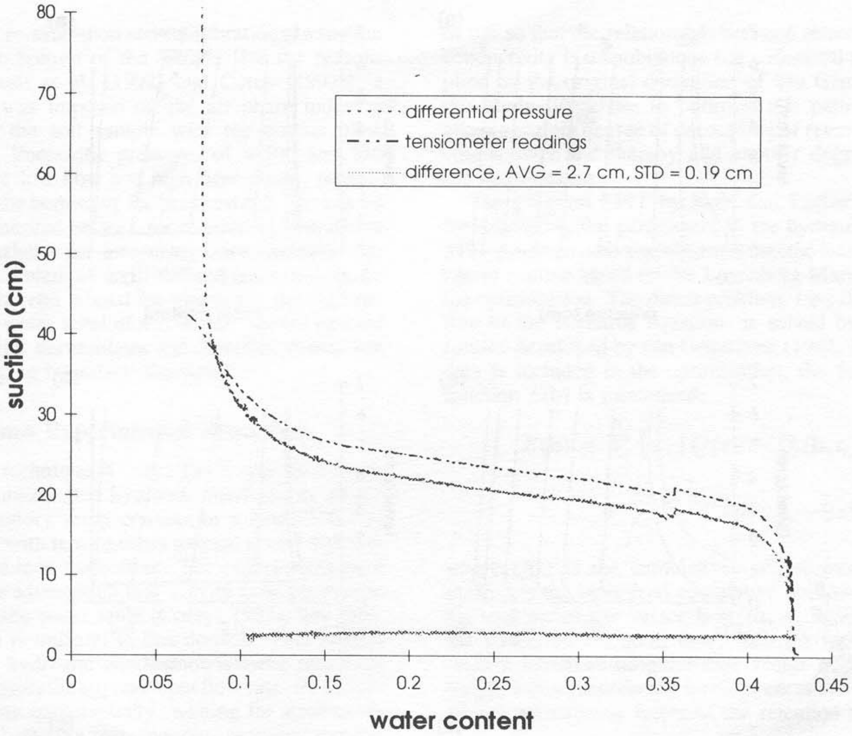

Fig. 3. Retention curves measured with both the differential transducer and with a tensiometer inserted in the pressure cell (no. 30 sand).

by the difference in hydrostatic pressure between the two measurement levels, thus supporting our assumption. The two sets of readings differ towards residual saturation because the tensiometer loses contact with the soil. Also, the readings obtained by the syringe pump method are questionable in this measurement range where the water phase no longer forms a continuous phase.

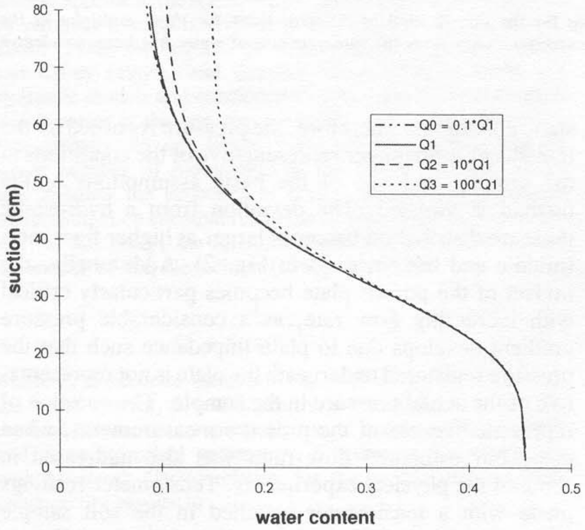

Fig. 4. Numerical simulation of retention curve measurement procedure at four different flow rates, sand no. 70; 0.1, 1, 10, and 100 times the actual experimental flow rate (Q = flow rate).

Water Content Distributions

A uniform water content distribution is observed at high saturations (early time). Nonuniformity in water content across the sample height occurs when the flat part of the retention curve is reached, where small changes in pressure are associated with large changes in water content. At the later stages, where the steeper part of the retention curve is reached and large increases in pressure are associated with only small variations in water content, the water content again approaches uniformity. The nonuniform distribution of water content occurring in the intermediate regime is most critical for materials with a relatively narrow pore-size distribution, which implies relatively large changes in water content with small changes in capillary pressure. Under such circumstances, the identification of a representative value for the water content corresponding to the imposed suction is difficult. This problem, however, is not specific for the proposed method but affects all methods operating at hydrostatic conditions. Note that the bubble pressure of the porous plate is exceeded at residual saturation ($t_{\max}$) due to the large pressure increase simulated, and the porous plate is thus drained.

The simulated impact of increasing the flow rate is illustrated in Fig. 4. Apparently, running the tests at too high a flow rate will cause the retention curve to approach an apparent residual water content that is unrealistically high. Similar results have been reported by Demond and Roberts (1991) who found that slowing the rate of discharge during the measurement of capillary pressure–

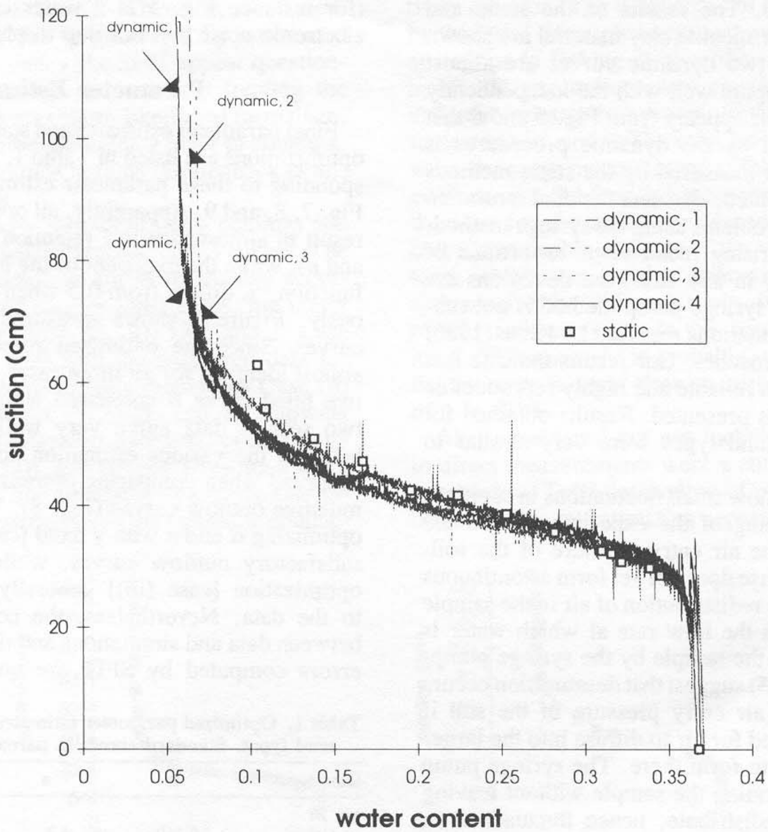

Fig. 5. Retention curves for sand no. 0. Four dynamic and one static measurements.

saturation relationships resulted in measurable decreases in residual saturation. Demond and Roberts (1991) suggested to explain this phenomenon by the increasing likelihood of entrapping water at the higher flow rates due to increased channeling and bypassing of pores. Apparently, the flow rate adopted in the physical experiments for no. 70 sand is adequate. Simulations on both finer and coarser materials showed similar satisfactory results.

The simulations clearly demonstrate that the withdrawal rate should be considered carefully, particularly when approaching residual water content, and a sufficiently low rate should be adopted to obtain valid results. We suggest to apply a numerical model based on rough estimates of the hydraulic properties to identify a proper or safe flow rate or to verify that a performed experiment was carried out at an adequate flow rate. Eventually, a data base will be available that provides safe flow rates for different soils and porous materials. In addition, the pump can be stopped occasionally (which can be done automatically by programing the pump) to check whether the measured pressure decreases notably when the pump is stopped. If that is the case, the applied flow rate is too high. Variable flow rates may be applied for different positions on the retention curve so that the intermediate range can be measured more rapidly than the extremes and the time required to obtain the retention curve can be even further minimized. The measurements in the vicinity of residual saturation should be used with caution because the syringe pump method is incapable of producing reliable results when the water phase is no longer continuous. In any case, this problem may not impact intended modeling efforts significantly because the flow for most practical purposes is negligible at water contents near residual. A possible advantage for future investigations would be to use a smaller pressure cell for the experiment, thus minimizing both the inaccuracies caused by the height of the sample and experimental time. In addition, alternative porous plates with higher conductivity and bubble pressure should be considered as capillary barrier at the water–air interface, minimizing the effect of plate impedance.

Syringe Pump Measurements

The results of the static and dynamic tests for no. 0 sand are shown in Fig. 5. The four dynamic tests are almost identical and they are in good agreement with the static results. Note that different samples were used for all the tests, and despite all precautions taken, it is generally not possible to obtain exactly the same packing from sample to sample. Thus the minor discrepancies between the curves seem to be caused by differences in packing. The static curve indicates that the flow rate used in the dynamic experiments is adequate; i.e., no noticeable drop in capillary pressure was observed when the pump was stopped. The results of the static and dynamic tests on the vermiculite clay material are shown in Fig. 6. Again, the two dynamic curves are almost identical, and they compare well with the independently measured static curve. It appears from Fig. 5 and 6 that the retention measured by the dynamic procedures is slightly lower than that measured by the static methods for both materials presented, also near residual saturation where we anticipated problems using the syringe method. We interpret this surprising result to be an artifact of differences in packing; in any case, the deviations are small. Apparently, the syringe pump method is not subject to experimental limitations reported by Klute (1986) for other dynamic approaches. Our results indicate that the syringe pump yields reliable and highly reproducible results for the materials presented. Results obtained for the remaining seven sand types were very similar to those presented here.

Some of the curves show small fluctuations in capillary pressure in the beginning of the experiment when the pressure approaches the air entry pressure of the soil. At this stage, the air phase does not yet form a continuous phase and therefore the redistribution of air in the sample is slow compared with the flow rate at which water is being withdrawn from the sample by the syringe pump. Corey and Brooks (1975) suggest that desaturation occurs at suctions below the air entry pressure of the soil if sufficient time is allowed for air to diffuse into the larger pores and for bubbles to form there. The syringe pump is continuously desaturating the sample without leaving time for the air to redistribute, hence fluctuations in capillary pressures are observed. The fluctuations do not affect the retention characteristics measured at later stages, and this artifact was minimized when a lower flow rate was used in the initial stage of the experiment. Thus, the fluctuations were not considered when fitting parametric models to the retention data. The scatter observed at intermediate stages in some of the curves (for instance Fig. 5 at a water content of ≈0.26) is electronic noise and not due to physical phenomena.

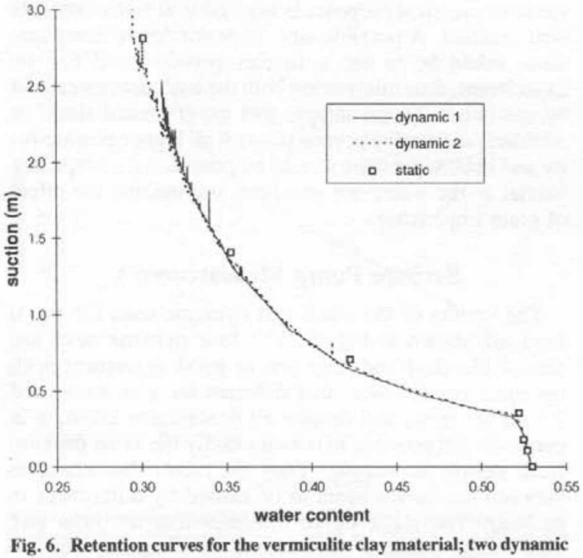

Fig. 6. Retention curves for the vermiculite clay material; two dynamic and one static measurements.

Parameter Estimation

Final parameter estimates and statistics for the different optimizations are listed in Table 1, and the curves corresponding to these parameter estimates are presented in Fig. 7, 8, and 9. Apparently, all optimization procedures result in almost identical retention curve parameters (α and *n*), while the exponent of the hydraulic conductivity function, γ, differs from 0.5 when optimized simultaneously. Figure 7 shows measured and fitted retention curves. Since the optimized retention parameters are almost identical for all three cases, only one representative fitted curve is compared with measured data. The two sets of data agree very well. A large difference between the various estimation approaches is however observed when comparing optimized and measured cumulative outflow curves (Fig. 8). The figure shows that optimizing α and *n* with γ fixed [case (ii)] does not yield satisfactory outflow curves, while including γ in the optimization [case (iii)] generally results in good fits to the data. Nevertheless, the correlation coefficients between data and simulations and the parameter standard errors computed by SFIT are quite similar (the only

Table 1. Optimized parameter estimates and statistics for the six sand types. Standard errors in parentheses.

Sand type†	α	*n*	γ	R^2
no. 30				
Case 1	0.069	4.2	0.5	–
Case 2	0.069	4.6	0.5	0.9994
	(0.0003)	(0.0591)	–	
Case 3	0.069	4.6	0.8	0.9994
	(0.0003)	(0.0594)	(0.0728)	
no. 70				
Case 1	0.028	6.0	0.5	–
Case 2	0.028	4.6	0.5	0.9993
	(0.0002)	(0.0750)	–	
Case 3	0.028	4.6	1.3	0.9995
	(0.0001)	(0.0657)	(0.2015)	
no. 125				
Case 1	0.026	7.0	0.5	–
Case 2	0.025	7.0	0.5	0.9994
	(0.0001)	(0.1348)	–	
Case 3	0.025	7.0	0.005	0.9997
	(0.0001)	(0.1020)	(0.0515)	
no. 00				
Case 1	0.025	7.7	0.5	–
Case 2	0.023	7.7	0.5	0.9935
	(0.0002)	(0.3299)	–	
Case 3	0.023	8.1	3.2	0.9998
	(0.0001)	(0.1465)	(0.1203)	
no. 0				
Case 1	0.026	8.0	0.5	–
Case 2	0.026	6.8	0.5	0.9946
	(0.0002)	(0.2695)	–	
Case 3	0.026	7.0	2.4	0.9981
	(0.0001)	(0.1763)	(0.1284)	
no. 1				
Case 1	0.044	12.0	0.5	–
Case 2	0.044	11.6	0.5	0.9976
	(0.0001)	(0.2949)	–	
Case 3	0.044	11.7	4.7	0.9990
	(0.0001)	(0.1986)	(0.3496)	

† Case 1: α and *n* estimated with RETC from retention data, assuming γ = 0.5; Case 2: α and *n* optimized from outflow and retention data with γ = 0.5; Case 3: α, *n*, and γ optimized simultaneously from outflow and retention data.

exception being the no. 125 sand). Therefore, such measures of the quality of the fit generated by automated parameter optimization codes like SFIT appear questionable. We recently extended our work to treating the estimation problem by a maximum-likelihood formalism, which allows a test of model adequacy prior to computation of parameter confidence regions (Hollenbeck et al., 1996, unpublished data).

Figure 9 shows directly measured (long column) and optimized unsaturated hydraulic conductivity curves. The results of Case (ii) estimation are in reasonable accordance with the measured data in some cases, but major deviations are seen for sands no. 30, 125, and 1. Case (iii) estimation agrees well with the measured data for all sands, particular improvements relative to Case (ii) are observed for sands no. 30, 125, and 1. Allowing the exponent of the unsaturated hydraulic conductivity function to vary [case (iii)], and thereby allowing a slight decoupling of retention and conductivity expressions apparently improves the estimation of the hydraulic conductivity function. The uniqueness of the estimates obtained by this semi-decoupled approach was tested heuristically analogous to Eching et al. (1994). Optimization of the hydraulic parameters was repeated four times with different (arbitrarily selected) initial estimates of the parameters α, n, and γ. The solution was considered unique when the program converged to the same final parameter estimates for the four optimizations. This was generally the case for the sands considered here (Table 2). The only questionable estimate is the value of γ for no. 125 sand, which converged to approximately the same final estimate for the different trials, but the standard error of this estimate exceeds the measure of the estimate itself. This anomaly is probably caused by the fact that the estimate is very close to zero, the boundary value of γ in SFIT.

Also considered but not included in the comparison to direct measurements were a couple of less successful approaches. Total decoupling of retention and hydraulic conductivity, by estimating separate van Genuchten pa-

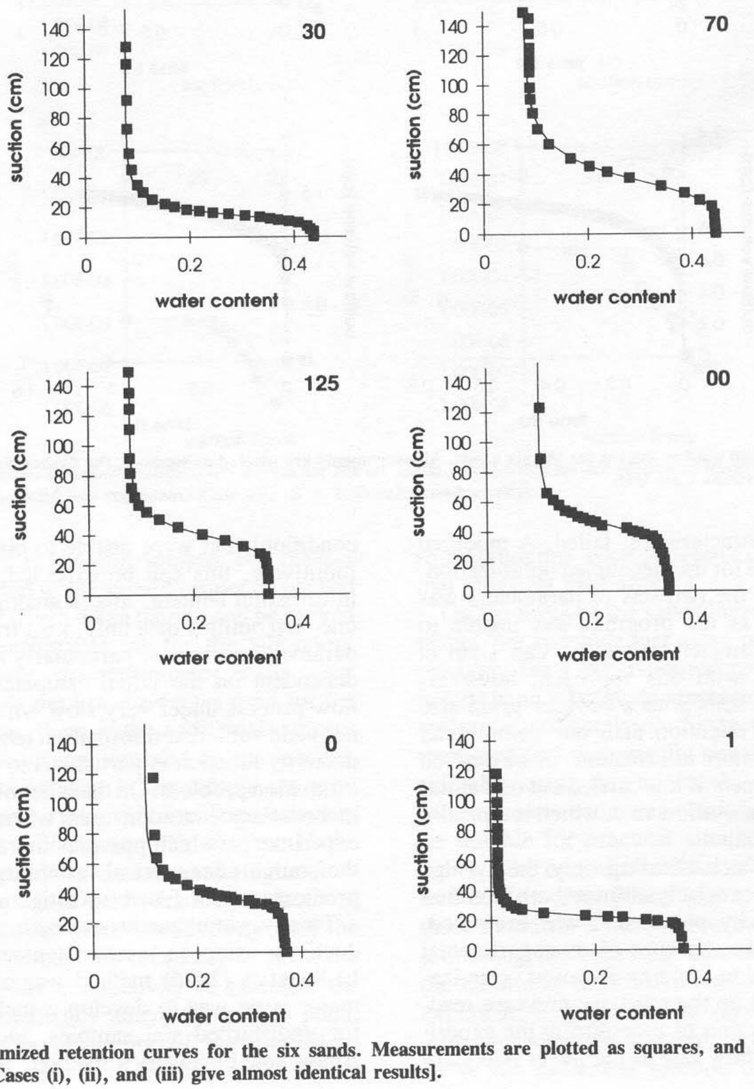

Fig. 7. Observed and optimized retention curves for the six sands. Measurements are plotted as squares, and the solid lines represent the optimized parameters [Cases (i), (ii), and (iii) give almost identical results].

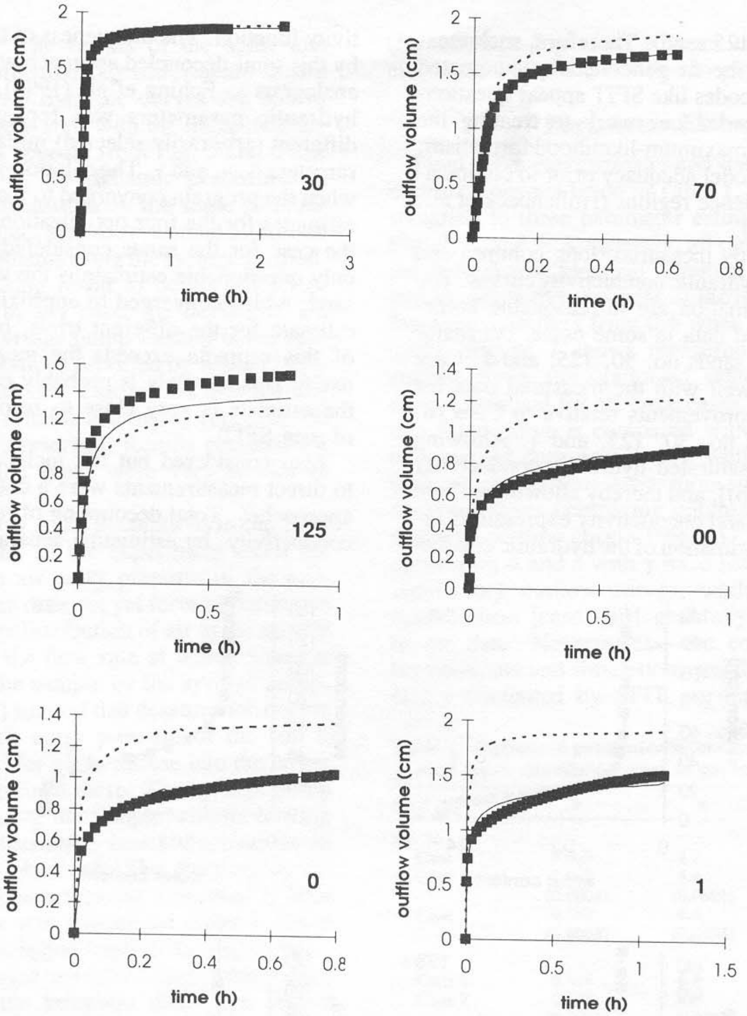

Fig. 8. Observed and optimized outflow curves for the six sands. Measurements are plotted as squares, the dashed lines represent Case (ii) [and (i)] and the solid line represents Case (iii).

rameters for the two characteristics, failed. A modified version of SFIT was used for this decoupled optimization. The total decoupling of the two sets of parameters was found to be ill-posed, as the program was unable to converge to unique parameter estimates. Van Dam et al. (1992) had success with this approach; however, they considered natural soils with a broader grain-size distribution and higher retention than our commercial sands. It is possible that more information can be derived from the outflow experiment if it is carried out on natural soils at a relatively slower outflow rate, which is typically the case for finer materials.

In addition, the approach of using only the syringe pump withdrawal data to inversely estimate both retention and hydraulic conductivity parameters was examined. This approach is attractive because of its experimental simplicity but was found to yield an ill-posed optimization. First we optimized on the capillary pressure readings, measured as a function of time during the experiment (using the applied flow rate as the lower boundary condition), but were unable to obtain unique estimates. Intuitively, this can be expected, because in terms of information content, this is analogous to optimizing on one-step outflow data only, i.e., transient data only. The parameter estimates, particularly n and γ, were entirely dependent on the initial estimates. It appears that the flow process under very slow withdrawal of water does not yield sufficient information (about the hydraulic conductivity function in particular) to establish a well-posed estimation problem. On the other hand, the abrupt change in boundary condition used when performing one-step experiments (which provides more information) is often the source of numerical instability of the simulation, a predicament for future investigations.

Finally, simultaneous estimation of the hydraulic conductivity using an inserted tensiometer and Ahuja and El-Swaify's (1976) method was also considered, but a major issue was to develop a method that can be used for undisturbed soil samples, too. For application of Ahuja and El-Swaify's method, it would be necessary

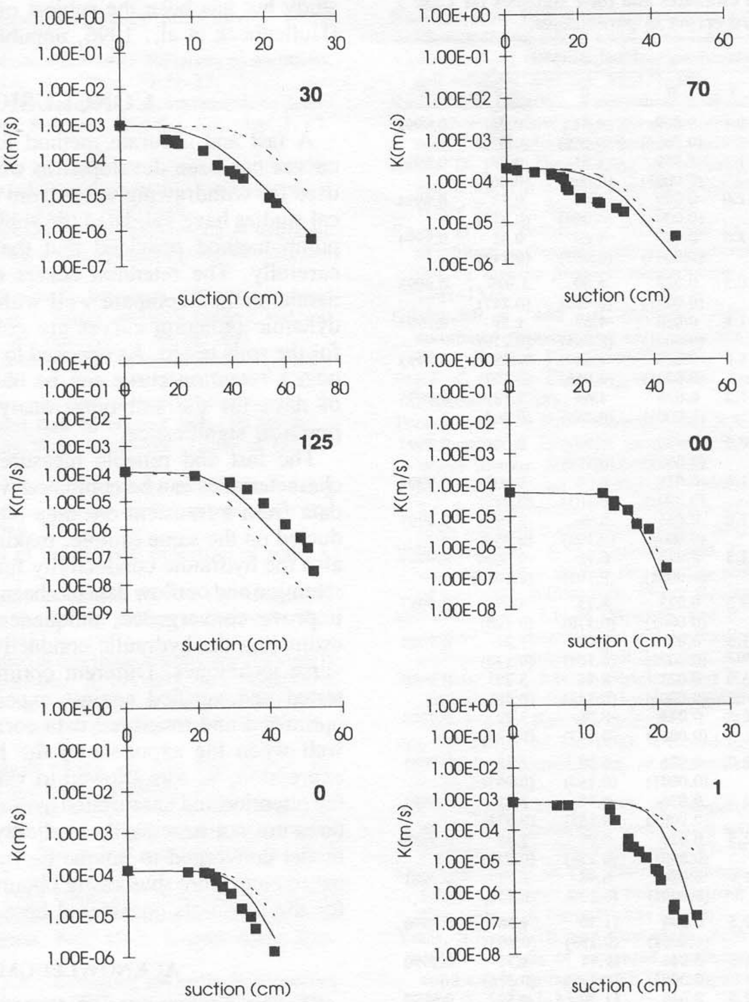

Fig. 9. Observed and optimized hydraulic conductivity curves for the six sands. Measurements are plotted as squares, the dashed lines represent Case (ii) [and (i)] and the solid line represents Case (iii) (K = hydraulic conductivity).

to insert a tensiometer in the soil sample, but contact problems can be expected when the soil cannot be packed around the tensiometer. Even when the soil can be packed directly around the tensiometer, as for the laboratory sands used in this study, the tensiometer easily loses contact with the soil. At the low flow rates used for the retention measurements, there is the additional concern that due to the low flow rates imposed on the sample, the hydraulic gradients existing in the sample are very close to zero. Hence, even a relatively small measurement error in the gradient will corrupt the results. If the experiment, on the contrary, is carried out at a higher flow rate to facilitate estimation of the hydraulic conductivity characteristics, the porous plate conductivity may be exceeded. Then it would become necessary to know the precise value of the plate conductivity to calculate the capillary pressure immediately above the porous plate. Because of the extreme sensitivity of the method to the plate conductivity, it would become necessary to re-measure this value between experiments as even minimal clogging may affect it.

On the basis of the sands investigated in these experiments, we conclude that using parameters estimated exclusively from the measured retention curve [Case (i)] results in an inadequate representation of the unsaturated hydraulic conductivity. Since the deviations between measured and numerically simulated outflow were considerable using this approach, one would suspect that the parameter estimates obtained in this case would not be suitable for practical simulations on a larger scale, either. The optimized values of α and n from retention and outflow data [Case (ii)] are almost identical with those found for Case (i). The important improvement in estimating the hydraulic conductivity and representing the measured cumulative outflow is obtained when the exponent, γ, is allowed to vary.

At last, we would like to point out the apparent instability of the numerical solution observed in some of the

Table 2. Initial and final estimates and their statistics for Case 3 optimizations. Standard errors in parentheses.

	Initial estimate			Final estimate			
Sand type	α	n	γ	α	n	γ	R^2
no. 30	0.059	6.8	0.8	0.069 (0.0003)	4.61 (0.059)	0.75 (0.073)	0.9994
	0.069	3.8	1.5	0.069 (0.0003)	4.61 (0.060)	0.79 (0.066)	0.9994
	0.029	7.8	1.0	0.069 (0.0003)	4.61 (0.060)	0.73 (0.213)	0.9994
	0.089	2.8	2.0	0.069 (0.0003)	4.62 (0.059)	0.82 (0.111)	0.9994
no. 70	0.034	4.4	0.5	0.028 (0.0001)	4.60 (0.065)	1.039 (0.147)	0.9995
	0.014	8.4	1.8	0.028 (0.0001)	4.63 (0.065)	1.39 (0.279)	0.9995
	0.054	6.4	1.0	0.028 (0.0001)	4.64 (0.053)	1.28 (0.197)	0.9995
	0.064	3.4	1.2	0.028 (0.0001)	4.64 (0.066)	1.32 (0.202)	0.9995
no. 125	0.026	7.0	0.5	0.025 (0.0001)	6.95 (0.102)	0.0001 –	0.9997
	0.046	5.0	1.5	0.025 (0.0001)	6.96 (0.103)	0.0051 (0.043)	0.9997
	0.066	3.0	1.0	0.025 (0.0001)	6.96 (0.102)	0.0045 (0.052)	0.9997
	0.036	4.0	2.5	0.025 (0.0001)	6.92 (0.101)	0.0064 (0.1089)	0.9997
no. 00	0.026	8.2	0.5	0.023 (0.0001)	8.13 (0.170)	2.89 (0.100)	0.9987
	0.046	6.2	1.5	0.023 (0.0001)	8.09 (0.161)	3.21 (0.130)	0.9988
	0.066	4.2	3.0	0.023 (0.0001)	8.23 (0.143)	3.27 (0.081)	0.9988
	0.036	2.2	2.5	0.023 (0.0001)	8.08 (0.147)	3.19 (0.120)	0.9988
no. 0	0.016	7.2	2.0	0.026 (0.0001)	6.89 (0.150)	2.18 (0.083)	0.9981
	0.046	3.2	1.5	0.026 (0.0001)	6.99 (0.163)	2.31 (0.074)	0.9981
	0.066	1.3	2.5	0.026 (0.0001)	6.99 (0.138)	2.38 (0.073)	0.9981
	0.036	8.2	1.0	0.026 (0.0001)	6.98 (0.176)	2.51 (0.128)	0.9981
no. 1	0.036	7.2	0.5	0.044 (0.0001)	11.70 (0.199)	4.58 (0.403)	0.9990
	0.056	5.2	1.5	0.044 (0.0001)	11.71 (0.199)	4.71 (0.378)	0.9990
	0.076	3.2	2.5	0.044 (0.0001)	11.70 (0.198)	4.59 (0.286)	0.9990
	0.046	9.2	1.0	0.044 (0.0001)	11.71 (0.199)	4.78 (0.349)	0.9990

simulated outflow curves. We attribute this scatter to problems in the numerical code and its time-stepping scheme. Finite-element approximations have been shown to display undershoot fronts and mass balance errors (Celia et al. [1990] as discussed in Ségol [1994], chapter 5). Such inaccuracies in the numerical solution may affect the inversion solution, even if they are small (mass balance errors were on the order of up to 5%). Another concern with SFIT is its automatic assignment of weights to the observations in the objective function, depending solely on the data's absolute magnitude. There is a limited option to manipulate the weights manually, but previous applications of SFIT in the literature have not been based on statistical argument in their choice of weights. We have continued this tradition in this study for the sake of comparison; however, it is our impression that estimation based on maximum-likelihood principles will be advantageous. Such investigation is beyond the scope of this study but has been the subject of our ongoing research (Hollenbeck et al., 1996, unpublished data).

CONCLUSIONS

A fast and accurate method for measuring retention curves has been developed in which a syringe pump is used for withdrawing water from a soil sample. Numerical studies have validated the applicability of the syringe pump method provided that the flow rate is selected carefully. The retention curves obtained with this dynamic method compare well with static curves, and the dynamic retention curves are convincingly reproduced for the soils tested. As opposed to traditional static methods, a retention curve can be obtained on a time-scale of days for the soil types analyzed, an advantage of practical significance.

The fast and reliable measurement of the retention characteristics can be combined with cumulative outflow data from a transient one-step outflow experiment conducted on the same sample, making it possible to derive also the hydraulic conductivity function. Combining the retention and outflow data has been shown to considerably improve convergence, uniqueness, and stability when estimating the hydraulic conductivity parameters by inverse techniques. Different optimization schemes were tested and verified against experimental data and the optimized and measured data corresponded particularly well when the exponent of the hydraulic conductivity expression, γ, was allowed to vary. This indicates that the retention and unsaturated hydraulic conductivity functions are not necessarily explicitly related. The inverse model converged to unique estimates and all hydraulic parameters were thus easily obtained in a matter of days for the materials considered here.

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