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A Glimpse into How Students Solve Concept Problems in Rigid Body Dynamics

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Overview

An examination of typical textbooks for the standard sophomore-level engineering dynamics course reveals that the vast majority of homework and example problems are quantitative in nature. They ask the student to find the percentage of energy lost…; determine the distance traveled…; calculate the normal force…; compute the maximum velocity… In teaching engineering dynamics, we focus on a systematic problem-solving process which will allow students to answer such questions. As characterized by McCracken and Newstetter[1], dynamics problem-solving is a process of transforming a problem into a series of different representations. In the textbook, the problem starts out in textual and pictorial form. The student is to transform this into a diagrammatic representation in the form of a free body diagram, and then into a symbolic representation which can be manipulated mathematically to solve for quantities of interest. Throughout the semester, we apply this process to problems using Newton's Second Law directly; using the work-energy principle; and using the impulse-momentum principle. We apply it to bodies that can be treated as particles, as systems of particles, and as rigid bodies with rotational inertia. Normally, we say that a student succeeds in the course if she or he can successfully apply this problem solving process to a series of quantitative exam questions similar in nature to the homework problems from the textbook.

In his book, Eric Mazur[2] describes an evolution in his teaching of introductory physics. Initially, he describes being very confident in his teaching: "… my students did well on what I considered difficult problems, and the evaluations I received were very positive." However, after reading articles by Halloun and Hestenes[3, 4], Mazur decided to test his students with a series of multiple choice concept questions. To an expert, the qualitative concept questions tend to look much simpler than the typical textbook questions; the need to perform calculations had been stripped away, leaving only a qualitative application of the concept. Some concept questions asked students to predict straightforward, but perhaps counterintuitive, consequences of Newton's Third Law. Other concept questions required students to think through a problem more deeply, but a simple free body diagram and direct application of physical principles would yield the appropriate answer. To Mazur's surprise, his students scored significantly worse on the concept questions compared to the quantitative problem-solving questions that he had written into his homework and exams

My own experience with using qualitative concept questions in engineering dynamics has been very similar. When I started using the Dynamics Concept Inventory[5] in 2006, students were performing poorly on such concept questions, even those students who did well on the traditional quantitative problem-solving homework and exam questions. The following year, I explicitly began covering qualitative problem-solving with concept questions as a formal part of the course. This activity usually took place during a weekly recitation section where the class size was smaller and it was more convenient to run a *Think-Pair-Share*[6] activity, not all that different from Mazur's *Peer Instruction* pedagogy. When I ask students to explain their answers to the qualitative concept questions, I was astonished to discover that students rarely drew free body diagrams. They ignored the physical principles discussed in class and, instead, relied on their own physical intuition. Mazur[2] reported something similar when he recalled a student asking, "… how should I answer these questions? According to what you taught us, or by the way I *think* about these things?"

During the recitation sections, starting in 2007, we would discuss how the systematic problem-solving process, including free body diagrams, could be applied to the qualitative concept questions as well. Over the semester, I observed what appeared to be a tangible improvement in students' reasoning through concept questions, and I witnessed a significant improvement in concept test scores. In 2010, I witnessed another significant improvement in concept test scores when I incorporated a game-based simulation environment called Spumone[7] into the course.

Despite these gains, however, scores on qualitative concept problems remain well below those of the conventional problem-solving. Running out of concrete ideas to improve students' conceptual understanding further, I recently conducted a small experiment to investigate their approach at solving concept questions. The experiment is the subject of this paper.

Going into the experiment, I hypothesized that students were still deferring to their gut instincts to solve the concept problems. Unlike the quantitative conventional problems, students are not required to show their work on the concept test, and there is no partial credit. Because there are many concept questions on the test and time is limited, I hypothesized that students only use a free body diagram, for example, if they believe that it provides useful and efficient approach to determining an answer. Otherwise, I hypothesized, they revert to their inner mechanical instinct.

To test the hypotheses, we recorded students verbalizing their thoughts as they worked through two sets of concept problems. Results are described herein.

Class Background

In the fall semester of 2014, a total of 50 students completed my engineering dynamics course and provided consent to use their data for research purposes. Figure 1 shows a scatter plot of students' test scores. The horizontal axis shows the average of students' scores on the six problem-solving questions they encountered on their midterm and final exams. The vertical axis shows the average of students' scores on qualitative concept test questions. There were a total of 53 multiple choice concept questions on the final exam. Some of the questions came from the Dynamics Concept Inventory. Other concept questions came from a pool that I have developed over the past several years. The fact that some students are indicated with a red x will be described in the next section.

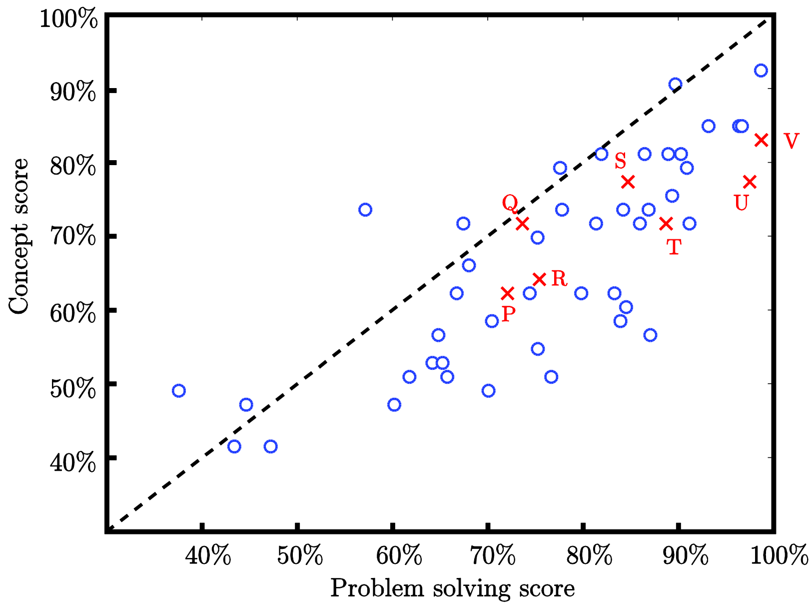


Figure 1. Scatter plot showing students' test scores on problem-solving tasks and concept questions.

As one can see from the scatter plot, the concept and problem solving scores are rather well correlated: $r = 0.793$, $p < 0.001$. As is apparent in the figure, there are a few students who scored near the dashed line, indicating that they scored nearly the same on the quantitative problem-solving and on qualitative concept problems. However, a large majority of students scored significantly lower on the concept questions as indicated by the large cloud of score pairs below the dashed line. On average, the difference between concept and problem solving was just over 10% lower ($T = 7.783$, $p < 0.001$), a full letter grade difference in my class.

Now one may argue that one should not compare the two scores since they were determined by two entirely different grading schemes. On the problem-solving questions, for example students were able to earn considerable amount of partial credit, even if they did not end up with the correct answer in the end. In contrast, scoring on the concept questions is all or nothing. Full credit is given to correct answers; zero credit is awarded for wrong answers. However, before one argues that that the concept scores are deflated, relatively, by the scoring mechanism, it is worth noting that it is common for students to earn full credit on problems which they answer correctly, but have an incorrect justification for choosing the answer. We will see examples of this later in the paper. Thus, it is entirely possible that the grading scheme for the concept problems overestimates students' understanding of the concepts. One thing that is clear from talking to the students is that they generally feel that the concept portion of the final exam is more challenging.

Participants

In selecting participants for this study, I chose 10 students at random from a larger pool of 16 students. The original pool of 16 students was chosen based on the following. (1) On midterm exams, they consistently applied the problem-solving methodology correctly. (2) They answered at least one problem-solving midterm exam question perfectly or nearly perfectly. (3) They perfectly (or nearly perfectly) answered a homework problem (see Appendix A) that covers the concept tested in this study. Of the ten students selected, two chose not to participate and one was not able to participate due to a scheduling mix-up. The remaining seven students who did participate are each indicated with a red 'x' in Figure 1, and labeled with letters P through V. Participants selected were also told to work through an online example problem for which solutions were posted.

Due to the small number of female students taking the course, I am not reporting how many women were included in the original pool of 16 candidates, or in the subsequent group of 7 participants. I will use the male pronoun when referring to individual students even though the student might be female. To do otherwise, I feel, would jeopardize the anonymity promised to students as a condition of their participation.

Because of the way that the initial pool of students was determined, it is not surprising that the set of participants come from the top performers in the class. This was intentional. I wanted to investigate how students who are capable of solving problems through the systematic problem-solving process try to solve qualitative concept problems.

Participants for the study were selected two weeks before the final exam, so their final concept test scores were unknown at the time. As was typical, it turned out that all seven students performed worse on their concept test, compared to the average problem-solving score. On average the difference between scores was 11.9%.

Methods

Within one week of the final exam, each student participant signed up for an hour-long session in which they were given two multiple choice concept problems to solve. Although the sessions were scheduled to last an hour, participants were told that they would probably need much less time to complete the task, and that they could leave when finished. Each participant was compensated with a gift card worth ten dollars.

Sessions took place in a small office with a desk, and a door that remained shut. Participants were given a few moments to read the questions; then they were asked to explain what they were thinking as they attempted to answer the questions. Participants' voice and writings were recorded with a Livescribe Echo™ smart pen.

During the sessions, the only other person in the room with each participant was a senior level undergraduate student whose primary job was to check that the recording equipment was functioning correctly and to make sure that the participants kept communicating their thoughts as they worked on the problems. Participants were told that the instructor of the course would not see their responses until after the semester had ended and grades had been posted.

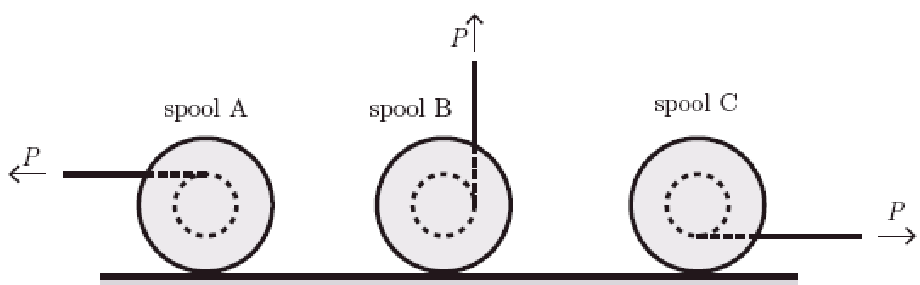
Results: Concept Question 1

Problem Statement. The first problem that the participants worked on is the three part problem shown in Figure 2. It is a problem in which a string is wrapped around the inner cylinder of a spool that initially sits motionless on a table. When the string is pulled, the problem states that the spool begins rolling without slipping. In which direction does the spool roll?

This is a classic problem that I usually share with my engineering dynamics class as a learning exercise. In my experience, students often answer the first two parts correctly, stating that the spool rolls to the left. However, they often mistakenly claim that spool C rolls to the left too.

1. The figure below shows three spools initially resting with zero velocity on a table. Each of the three spools has a string wrapped hundreds of times around an inner cylinder, in the counter-clockwise direction as shown in the figure.

   In each of the three cases shown below the string is pulled *gently*, in the direction shown. When the strings are pulled, the spools begin *rolling* on the tabletop *without slipping*. Assume that the string does not slip off the inner cylinder.



   (a) In which direction does **spool A** begin moving?
   A. To the left. B. To the right. C. Neither. It cannot move left or right.

   (b) In which direction does **spool B** begin moving?
   A. To the left. B. To the right. C. Neither. It cannot move left or right.

   (c) In which direction does **spool C** begin moving?
   A. To the left. B. To the right. C. Neither. It cannot move left or right.

Figure 2. First problem used in this study to examine students' approaches in answering qualitative concept questions.

diSessa[8] describes the misunderstanding in this problem in terms of phenomenological primitives or p-prims. A p-prim is a simple explanatory or descriptive idea about a physical phenomenon that is accepted by a person uncritically[9]; it is built upon a lifetime of observations. In this particular problem there are two p-prims at work. One is the “Force as a mover” p-prim that asserts: “Pushing an object from rest causes it to move in the direction of the push.” The other is “Force as a Spinner,” which is similar to the previous p-prim except that it derives from experiences pushing an object off-center and causing it to rotate. In part (a) of the problem, both p-prims lead to the same result: motion and rotation to the left. In part (b), students are told that the spool rolls without slipping. Since moving upward is not an option, students fall back on the “Force as a spinner” idea and state that the spool rolls to the left. In part (c), though, the two p-prims contradict. “Force as a Mover” asserts that the spool should move to the right whereas “Force as a spinner” dictates that the spool move to the left. According to diSessa, most novices choose the spinner p-prim and therefore would predict that the spool rolls to the left.

Engineering students who knows Newtonian mechanics, however, should be able to put aside intuition and be able to reason through questions similar to that shown in Figure 2, basing arguments on physical principles. In particular, one should be able to recognize that if one considers moments about the point of contact, then in the first two cases, the net moment is counter-clockwise and the spool must roll to the left. In the third case, the moment about the contact point is clockwise and the spool must rotate to the right. It is the exact same approach all 7 participants used when they calculated the acceleration of a solid cylinder as it rolled without slipping down a ramp (Appendix A). Could the students apply the same principle in a context which does not require a numerical calculation?

Students' Responses. None of the seven students had a particularly satisfactory response to concept question 1. The amount of time each student spent on the first question is tabulated in Table 1. Students' approaches are outlined below.

Table 1. Amount of time that students spent on concept question 1, all three parts.

Student		P	Q	R	S	T	U	V
Time (min:sec)		1:20	5:20	8:30	1:33	2:20	9:00	8:50

Student S and *Student T* had similar approaches to the concept question. Neither of them drew a free body diagram (FBD), even though they always drew FBDs on problem-solving questions they encountered on their midterm and final exams. For parts A and B, they simply observed that tension from the string creates a counterclockwise moment (without indicating which point the moment about). Therefore, they argue, the spool rolls to the left. Both students also perceived a counterclockwise moment for part C. Again, they did not mention what point they're taking moments about, presumably the center. But they also recognized that this observation of leftward rotation was in conflict with the fact the fact that the string is pulling toward the right. *Student S* reconciled this by stating that the block does not move left or right, without any further justification. After recognizing the conflict, *Student T*, simply stated the string “cannot drag the spool along with it”, so therefore, it must move to the left. Note that the final determination was not based on a physical, but rather a hunch.

Student P did draw a free body diagram. However, it only included one force, that of the string. Based on his previous work, I find it difficult to believe that *Student P* truly believed that the string tension was the only force. Instead, it seemed like a quick shortcut. However, based on the incomplete FBD, he stated that there was a counterclockwise moment *about the center*. Therefore, the spool starts to rotate to the left. He started drawing FBDs for parts B and C, but then stated that the rationale and answer was the same for the other two cases as well.

Student Q was interesting to observe. It was clear that he wanted to answer the question using his intuition: “It’s the natural thing to do. But every time I use my intuition, I’m wrong.” Therefore, *Student Q* went on a hunt to find a justification. He started drawing an FBD, with only a tension force. He wrote down an appropriate moment equation, but then got flustered. [He was the only one who wrote down a moment equation.] Finally, *Student Q* focused on the fact that in all three cases the string is wrapped around the inner cylinder in the counterclockwise direction. And since, “when you pull the string, the spool must unwind,” all three spools must rotate to the left. Of course, this unwinding principle is not based on physics and it is not true.

From Figure 1, the reader will notice that *Student U* and *Student V* had the highest test scores among the seven participants, having almost a perfect score on the problem-solving exam questions. Interestingly, these two students had very similar (flawed) approaches. Both students drew perfect FBDs with all appropriate forces. However, both also skipped over the physical principles and simply stated that the spool begins rotating counterclockwise, which is only correct in two of the three cases. There was no mention of moments or angular momentum principles. Instead, both started writing kinematic equations relating velocities of different points on the rigid body to each other, using the angular velocity. *Student V* performed the kinematic calculations correctly and found that all three cases moved to the left. *Student U* made mistakes in the kinematic calculations and obtained different answers in the end.

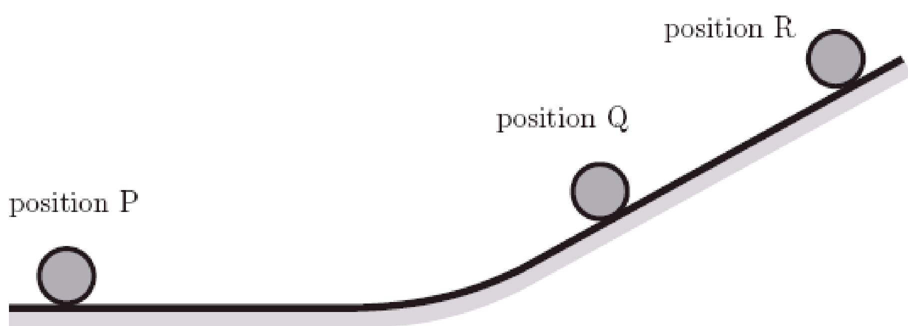
Student R had the most interesting response, in my opinion. His FBD contained all relevant forces, except for the friction. There were many things that *Student R* said that were not correct. However, he seemed to see the big picture more than the other students did. In part (a), he noticed that the only horizontal force was to the left; therefore, the spool must accelerate to the left. Similarly, in part C, he noticed that all horizontal forces were to the right, so the body must accelerate to the right. Of course, he omitted the critically important friction force in the system. Without friction, the spool would not rotate. Nonetheless, he used an appropriate physical principle in his reasoning. *Student R*’s argument for case B was a little more tortured and self-contradictory, but he selected the correct choice in the end.

Results: Concept Question 2.

Problem Statement. The second concept problem is motivated by a problem-solving question that I put on the final exam in spring 2014, the previous semester. Then, I had asked students to find the force of friction acting on a cylinder rolling up a slope. At the beginning of the problem, it appeared that nearly all students originally drew the friction force on the free body diagram in the wrong direction. However, when confronted with inconsistencies in their problem formulation, about a third of the students, typically the higher performing students, corrected their original assumptions.

The second concept question we use in this study is motivated by that previous experience and is shown in Figure 3. In this problem, I simply ask students which direction the friction should act.

2. Initially, a cylinder is rolling on a horizontal surface to the right (position P). When it encounters a slight hill, the cylinder begins slowing down and reaches an ultimate height at position R before turning around and heading back down hill. At position Q, the cylinder is still rolling up the hill. Over the entire path from position P to position R, the cylinder *rolls without slipping*.



When the spool is at position Q, rolling up the hill, in which direction does friction from the hill act on the cylinder?

A. Up the hill, to the right B. Down the hill, to the left C. Neither. There is no friction.

Figure 3. Second problem used in this study to examine students' approaches in answering qualitative concept questions.

Here, if one considers moments about the center of mass of the cylinder, one finds that the only force which contributes to the moment is that of friction. Since the object is traveling uphill, rolling without slipping, the clockwise rotation rate must be slowing down. Therefore, the friction must be uphill and to the right in order to cause the correct *angular acceleration*.

In the example problem (see Appendix A), which all students claimed they worked through, we similarly had to take moments about the center of the bowling ball so that we could quantify the effect of friction on rotation. Did any students apply this approach – or other suitable approach – to the concept question?

Students' Responses. Three students picked correct direction, but none of them had an appropriate justification for their choice. Comparing Table 2 to Table 1, we see that *Students Q*, *R*, and *U* spent significantly less time on concept question 2, compared to the first question. The others invested roughly the same amount of time. An outline of students' approaches is outlined below.

Table 2. Amount of time that students spent on concept question 2.

Student		P	Q	R	S	T	U	V
Time (min:sec)		1:10	1:45	5:20	2:25	2:08	0:25	8:30

Student S worked in fits and starts. He started with a verbal explanation of the answer. When he got stuck, he started drawing an FBD. Then he declared that rolling without slip means no friction. Next he started drawing an FBD at point P. Without resorting to any physical principle, he declared the friction force at point P was to the left and extended the result to point Q.

According to *Student U*, cylinder is rotating clockwise as it travels uphill. For that reason, the friction acts downhill. That's it.

Student P decided to take work-energy approach to the problem. He drew diagrams, but they weren't free body diagrams. The diagrams mostly indicated direction of motion, which can be useful in work-energy problems. Then he observed that when the cylinder is traveling uphill, it is slowing down. Therefore, he concluded, friction does negative work. Again, *Student P* is making a connection to a physical principle in a mostly meaningful way. However, he missed important contributions to the work due to sloppiness. It is the gravity causing the negative work. He knows how to make FBDs. He just chose not to.

Student Q and *Student T*. They observed that as the cylinder rolled up the hill, it was rotating clockwise. Because of the rotation, they argued, the point on the cylinder in contact with the ground would be moving to the left if not for the friction. Because of this, they argue, the friction is up and to the right. It is a kinematic argument.

Student R and *Student V* both drew perfect FBDs with all relevant forces. Both noticed that the friction force is the only force which creates a moment about the center of mass. However, they misapplied the physical principle. *Student V* attempted to relate the moment to the angular velocity, which happened to be in the opposite direction of angular acceleration. *Student R* related the friction to acceleration of the cylinder which he mistakenly said was uphill.

While *Students R*, *Q*, and *T* selected the correct answer, none had answers that were even close to satisfactory.

Discussion & Reflection

Each time I try to look in depth at what students are actually learning, or not learning, in my classes, I am humbled. I have to relearn how difficult a job this is. If we were to only look at the answers that students provided to the multiple choice concept questions, collectively, they got 18 of the 28 questions correct. That is 64%. However, looking more carefully at the student's responses, I find it hard to justify giving students any credit. The students did not provide what I would consider proper justification to any of the correct answers.

To be clear, these students are not dumb. In fact, two of the students in this particular group received nearly 100% credit on the six long-form problem-solving questions that they were asked to complete as part of their midterm and final exams. The rubric for scoring those exam questions weights the problem-solving *process* more heavily than the correctness of the final answer. Therefore their high scores on their exams indicate that they could draw perfect FBDs; they could choose an appropriate physical principle to use; they could correctly apply the principle to derive equations of motion; they could solve the equations for the quantities of interest; they could verify the units of the answers; and they could interpret the results. All seven students could do it. Yet, in the concept problems studied here, it is the systematic *process* of thinking through a problem, of taking into account all relevant forces (FBD), of connecting the phenomenon to a physical principle, and of isolating the quantity of interest that was lacking.

One thing that this study demonstrates is that qualitative concept questions are fundamentally different, in the minds of the students, than the customary quantitative problems one typically finds in textbooks. We experts see both types of questions as being part of the same thing. We apply the same physical reasoning to both. In fact, we may consider the concept questions to be easier since we do not have to perform calculations. Dynamics students approach these problems very differently.

A quick review of the literature shows that the phenomenon of students neglecting their formal training when trying to solve physics problems that are somewhat different than the classical textbook problems they are accustomed to is not uncommon[4, 10-12]. The observations are consistent with Redish's[13] *Resource Model* of how people learn physics. The Resource Model is built from ideas that have mutual agreement in the neuroscience, cognitive science, and behavioral science communities, but is sufficiently course-grained to apply to the messy process of learning that occurs inside and outside of classrooms. In this model, a "resource" is a knowledge element or collection of knowledge elements that are linked together by cognitive construction. The resources exist in long-term memory. When transferred to working memory, they are easily processed by the individual as a seeming coherent manifestation of truth. Knowledge structures are the connections between knowledge elements that form a resource. Although some have used the term schema, Redish uses "knowledge structure" to emphasize that such structures are often in a state of flux for novices.

Resources and knowledge structures of experts are relatively complete, connected, highly organized, and internally and externally consistent. This organization allows, for example, experts to see that the concept questions and the traditional quantitative problems as being the same. In contrast, the resources and knowledge structures of novices are smaller and more fragmented. They may be internally and/or externally contradictory. Because of the fragmented nature of their knowledge structures, novices may not see the relationships between the two types of problems measured by the two axes in Figure 1.

Another important distinction between experts and novices, for our purposes, is that of *control*. This is the process by which individuals activate (or elicit) specific resources in certain contexts. Again, experts have highly refined control mechanisms which effortlessly activate resources that are appropriate and helpful in the context of the problem being solved. Novices are less able to recognize context and elicit appropriate resources.

A few years ago, when I began experimenting with concept problems and concept inventories in dynamics, I conducted an informal, simplified version of the study described in this paper. Back then, the gap between students' perceptions of concept problems and more traditional quantitative problem-solving tasks was much more stark. Those I interviewed back then, the idea of applying the systematic problem-solving approach to the concept problems didn't even cross their minds. Back then, performance on the concept inventory was only slightly correlated with exam performance.

This time was different. Even the participants who struggled most in the current experiment showed elements of self monitoring in which they tried to suppress their gut intuition in favor of a more objective justification. In his session, *Student Q* stated: "[using intuition] is the natural thing to do. But every time I use my intuition, I'm wrong." *Student S*, and *Student T* expressed similar sentiments, although they struggled with how to do it. *Student P* tried to couch his explanations in terms of physical principles but neglected to apply a systematic technique for cataloging the forces so that one could account for all relevant effects. *Student U* and *Student V* generated great free body diagrams, expressed kinematics principles mathematically; they just didn't express the physical principles appropriately. *Student R* was the only one who expressed a physical principle with sufficient rigor; he just did not recognize that is approach was flawed because he messed up the FBD. Since that earlier informal experiment, there has been substantial improvement in how my students perceive and approach qualitative concept questions.

As stated earlier in the paper, there has also been a significant improvement in my students' concept test scores over the years, part of which has been documented[7] in the literature. Although the current study casts some doubt on the validity of concept test scores, the fact that concept scores have become much more correlated with traditional problem-solving scores suggests that something positive is happening.

The biggest take-away of the current study, though, is that there is still enormous progress that can be made. Despite the improvement, my students are still performing poorly on the concept questions. Based on listening to students struggle through the problems, the connection between qualitative concept problems and the systematic problem-solving process still seems tenuous. I hypothesize that my students think of physical principles as equations to be computed. They don't interpret them as rules that can be applied qualitatively as well. In the months and years ahead, I look forward to exploring this further, and developing instructional strategies to close the gap.

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Appendix A

Homework problem that all participants solved nearly perfectly before participating in the study.

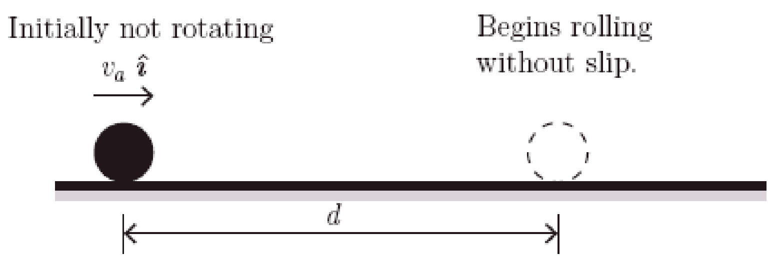
The figure on the left below shows a block of mass m that is released from rest on a ramp at angle θ. After it is released, the block begins sliding without friction.

The figure on the right shows a uniformly solid circular cylinder with the same mass m and radius R released from rest on a ramp with same angle θ. After it is released the cylinder begins rotating without slipping.

1. Find the acceleration of the block on the left after it is released.
2. Find the acceleration of the center of the cylinder after it is released. As you are solving the problem compare the relative advantage/disadvantage of taking moments about the center of the cylinder and about the point of contact with the ramp.
3. Compare the accelerations of the two systems. Explain why they are the same or different.

Example problem that all participants worked on before participating in the study.

When you "roll" a bowling ball, you may notice that the ball starts off sliding down the lane. Furthermore, if you don't do any crazy spins or roll it too fast, you'll notice that the ball begins rolling without slipping at some point before it hits the pins.



Suppose a bowling ball of mass m and radius R initially has speed v_a and is **not** rotating. Given these three quantities and the coefficients of static (μ_s) and kinetic (μ_k) friction, determine, the distance d that the ball travels before it begins to roll without slipping.