Spatiotemporal cooling of electronics using latent energy might be achieved by closely spaced, rapid departure of small bubbles. One means to achieve small diameters during boiling is to provide an additional upward force during bubble formation, such as that from vapor extraction. Experiments were conducted of bubble extraction using constant flow rates of both air and vapor that ranged from 30 to 90 mm$^3$/s. Extraction was achieved with a hydrophobic porous membrane sealed to a tube in which a vacuum was drawn. The gap between the extraction and supply surface was varied from 0.5 to 3.25 mm. Only individual bubbles that ruptured at the top surface while still attached to the supply surface were considered. Bubble departure diameters are approximately 80% of the gap height. As with unconfined bubbles in pool boiling, the bubble frequency varies inversely with departure diameter. Correlations for bubble rupture, bubble departure, and bubble frequency are presented as a function of gap height. Using the three distinct regimes identified in the experimental study, namely, growth only, growth with extraction, and extraction only, an effective bubble diameter model and an appropriate static force balance were developed. These were used to predict bubble departure frequencies and diameters, respectively, under confined extraction conditions.

INTRODUCTION AND BACKGROUND

As noted by Hamann et al. [1], new demands are being placed on high-heat-flux cooling systems that are based on spatiotemporal cooling capabilities, particularly at very small scales. Consequently, the ability to provide localized cooling on demand is an area of increasing interest. Boiling heat transfer is one means of achieving high flux cooling, and by introducing artificial nucleation sites, bubble formation can be localized. If the bubble size can also be controlled, the spacing of artificial sites can be designed to avoid bubble interactions (Judd [2]). One means of controlling bubble size is to add an additional upward force that causes the bubble to depart at a smaller diameter. To create such a force, it is proposed to locate a hydrophobic, porous extraction surface opposite and parallel to the surface where vapor is being generated. The extraction surface is located such that it confines the bubble during formation, providing the additional force necessary to create smaller bubbles.

The application of vapor extraction to heat sinks was first introduced by Apreotesi et al. [3] using a fractal-like branching microchannel flow network and by David et al. [4] for several microscale configurations. The latter group extended these studies by employing a computational fluid dynamic analysis (Fang, et al. [5]). All results show that vapor extraction results in lower operating temperatures with corresponding lower channel pressure drops. As an added benefit to the potential of regulating heat transfer, extraction has the potential to reduce or alleviate flow instabilities in convective boiling heat sinks, as noted by Qu and Mudawar [6].

In designing microchannel heat sinks with extraction, it is necessary to understand the design constraints, which include, among other things, the advection to extraction rate ratios. Many of these constraints were identified, and recommendations made, by David et al. [7] for adiabatic and diabatic microchannels with advective flow. Xu et al. [8] identified design constraints for adiabatic microchannel flows.

In a model developed to assess pressure drop through fractal-like branching channels experiencing in situ vapor extraction, Salakij et al. [9] recognized the drawback of using the entire membrane area for predicting the amount of vapor being extracted. In the model, Darcy’s Law was used to predict extraction rates. Based on this need, Cappello et al. [10] studied the
extraction potential through a hydrophobic porous membrane using both water vapor/liquid and air/water mixtures with no advection, and determined that the area of the bubble in contact with the extraction surface is the crucial area to employ for proper application of Darcy’s Law. Fazeli et al. [11] studied two-phase gas and vapor extraction from a 1-mm gap with no advective flow. By increasing the pressure inside the gap, injected gas bubbles were observed to decrease in size and, in the case of vapor generation, the critical heat flux was delayed. Prediction of the bubble diameters during simultaneous growth and extraction is necessary to design an effective spatiotemporal cooling system driven by extraction. For the case of unconfined boiling, bubble departure diameters and bubble departure frequencies were investigated and semiempirically modeled in the early work by Ivey [12]. Numerous studies followed resulting in several correlations, but that of Kocamustafaoğulları [13] is cited quite extensively, particularly in commercially available computational codes.

Static force balances are generally preferred as a means of predicting departure diameters; however, it is important that the forces relevant to the situation are included. Thorncroft et al. [14] provide a thorough assessment of forces acting on unconfined bubbles and detachment models for a variety of static and cross-flow scenarios.

The first goal of the present study is to experimentally examine the effects of bubble extraction on bubble dynamics by characterizing departure diameters and departure frequencies under confined extraction. Three regimes are identified in the process: (I) growth only, (II) growth with extraction, and (III) extraction only. These are shown in Figure 1. The second goal is to develop a theoretical model for predicting the bubble diameter as a function of time and confinement height, recognizing that it may need to be made semiempirical at this initial stage of development. Finally, given that forces contributing to bubble departure during confined extraction have not yet been studied, the third goal is to assess the relevant forces acting on a bubble over the course of its lifetime. Included in this assessment is the contribution of the exit momentum, which, unlike the inlet momentum, does not go to zero at bubble departure. Once determined, the relevant forces can be used in a static force balance to determine bubble departure diameter. This information is then used with the bubble diameter model to determine the bubble departure frequency. To improve the general applicability of the model, both adiabatic and diabatic cases are studied, using air/water mixtures for the former and heated bubble generation at an artificial nucleation site for the latter.

**EXPERIMENTAL FACILITIES**

The experimental test facilities for the diabatic study and the adiabatic study are shown in Figures 2a and 2b, respectively. Consistent between the two facilities are the test chamber, the extraction chamber, and the lighting configuration. The surfaces at which the bubbles were generated differ, but are generally called the supply surface. The test chamber, 64 × 54 × 50 mm (deep), consists of walls made of glass for viewing the bubble and an aluminum base in which the silicon surface for...
diabatic studies and the aluminum orifice plate for the adiabatic study are mounted. The chamber is filled with approximately 150 ml distilled water. For the diabatic case the water is degassed by vigorously boiling it for 30 min in the test chamber using two submerged cartridge heaters. The heaters are also used to maintain the pool temperature between 99.0 and 99.2 °C during testing, as measured with a K-type unsheathed thermocouple. The adiabatic study is conducted at room temperature.

The extraction surface, which confines the bubble, was formed from a hydrophobic PTFE membrane from Sterlitech with a specified pore diameter of 0.45 μm, porosity of 55%, and thickness of 15.6 μm. The membrane also has a laminated support layer. The membrane was secured at the bottom of a glass tube (4.2 mm OD, 2.1 mm ID), which was connected by tubing to a vacuum source to create a 35-kPa pressure differential across the membrane for the diabatic case and 14.5 kPa for the adiabatic case. The vacuum was created by suspending a free weight from two 10-ml syringes connected to the glass tube. The pressure inside the glass tube was recorded at 1000 Hz during testing using a differential pressure transducer (OMEGADYNE PX 409-015DWU).

The membrane surface diameter was larger than the bubble diameters being generated for all cases studied. The extraction surface can be positioned to create gap heights, \( H \), ranging from 0.52 to 4.0 mm, although in this study the maximum \( H \) value used is 1.9 mm for the diabatic case and a range of 1.2 to 3.25 mm for the adiabatic case. In addition, the confinement surface was removed for studying the unconfined bubble departure diameter and frequency.

For the diabatic case, an artificial nucleation site was machined in a 675-μm-thick silicon substrate using an ultraviolet (UV) laser (ESI 5330 UV Laser Microvia Drill) to be 30 μm in diameter and 100 μm deep. The nucleation site was heated to 81.4 W/cm² using a high-repetition-rate pulsed Nd:YLF laser (New Wave Research, Pegasus PIV, dual diode Nd:YLF 527 nm). The pulse rate and energy levels were controlled up to 10,000 pps per laser diode with a nominal pulse duration of 100 ns. The beam characteristics were evaluated at the bottom of the silicon surface by imaging the intensity profile using a CCD chip. Based on the paper by Aspnes and Studna [15], the laser beam at the silicon test surface had a theoretical 37.6% reflectance with 99% of the entering light absorbed in the first 17 μm. The reflectance corrected laser power was determined by mapping the intensity distribution. The equivalent laser beam diameter was defined based on the decay of intensity to 10% of the peak intensity. The laser spot size used for this study was 1.21 mm diameter, with 0.93 W delivered to the bottom of the silicon surface using a 3500-Hz laser pulse frequency. It was shown by Fox et al. [16] that these conditions resulted in a steady temperature at the top of the silicon surface to within 0.01% of the surface excess temperature during boiling.

For the adiabatic case, a 1-mm-thick aluminum plate with a 0.5-mm-diameter hole formed an orifice through which air was injected at a controlled rate using a programmable syringe pump (Cole Parmer). As designed based on the paper by Teraska and Tsuge [17], a constant volumetric flow rate was achieved by using a hydrophobic porous plug to produce the required pressure drop across the orifice. The three-phase contact line remains pinned at the orifice, as predicted by Dyson [18]. Design details are provided in Juarez [19]. A fixed nominal injection rate of air of 90 mm³/s was used for the adiabatic case.

Imaging of the bubbles was obtained using a high-speed, high-resolution (HSHR) camera (Phantom v5.0 with a Mitutoyo M Plan APO ×10 lens) for the diabatic case (resolution of 9.2 μm/pixel) and a Nikon J4 camera with a Nikon AF Micro Nikkor 60-mm lens for the adiabatic case (resolution of 36.9 μm/pixel). Images were acquired between 1,000 and 12,500 fps depending upon the conditions, and for gap heights between 0.52 and 3.25 mm. The chamber was backlit with a halogen light directed through a fiber-optic cable to a defusing sheet.
Data Reduction and Uncertainty Analysis

The images for each frame were processed to determine bubble size as a function of time from inception to full extraction. A Canny edge detection method was used to determine the extent of the bubble. Since only two-dimensional images were available, the bubble volume was found by taking one-pixel-high, horizontal slices through the bubble and assuming each slice to be axisymmetric to calculate the associated volume. All slices were then summed to get the total volume of each bubble for each frame. This volume was then converted to an equivalent diameter by assuming the total volume to be that of a sphere. To test the accuracy of the algorithm, a ball bearing of 7.15 mm diameter was imaged and the volume calculated was found to be within 0.5% and 1.7% of the actual value for the diabatic and adiabatic cases, respectively.

Uncertainty values for statistically averaged data, including bubble diameters, times, and frequencies, are reported in the results. However, manufacturer-reported, calibrated, and assumed uncertainties for use in propagation uncertainty analyses are reported here. The uncertainty in gap height is ±0.02 mm, whereas the uncertainty in a single measured bubble diameter is ±0.01 mm and ±0.02 mm for the diabatic and adiabatic cases, respectively. The uncertainty in time measurements is ±0.1 ms and ±0.42 ms for the diabatic and adiabatic cases, respectively. The calibrated uncertainty in pressure transducer is ±0.2 kPa, and the reported uncertainties for the thermocouple and syringe pump are ±1.1 °C and ±2.3 mm/s, respectively. The uncertainty in heat flux was determined to be ±1.7 W/cm². Assumed uncertainties used in propagation analyses include that of the membrane permeability, which is ±0.5 × 10⁻¹⁵ m² and ±1 × 10⁻¹⁵ m² for the diabatic and adiabatic cases, respectively, ±2.5 μm for the membrane thickness, and a ±3% uncertainty in the maximum area over which extraction is modeled.

Experimental Results

Because the source of the bubbles differs for the adiabatic and diabatic cases, the surface from which the bubbles originate is generically termed the supply surface. A series of high-speed movies were taken of bubbles that originate at a single site on a supply surface and then are extracted through the porous, hydrophobic confinement surface. The process is referred to as confined extraction. The distance between the two surfaces, referred to as the gap height, was the primary variable of interest. In reviewing the movies it was determined that, for the range of gap heights studied, there were three generic bubble types.

Type 1 bubbles reach the extraction surface and rupture before they depart from the supply surface. Type 2 bubbles coalesce with a previously departed bubble prior to departing from the supply surface. Type 3 bubbles depart from the supply surface prior to rupturing at the extraction surface. Type 1 bubbles are of the most interest because they are predominant at the small gap heights where the bubbles are observed to depart at small diameters and high frequencies. These are anticipated to be the most useful for spatiotemporal cooling and are further divided into individual events, as identified in Figure 3a. The events are (I) inception of the bubble, (C) contact of the bubble with the extraction surface, that is, a liquid film exists between the bubble and surface and the bubble has deformed, (R) rupture of the liquid film separating the bubble from the extraction surface, which is accompanied by an instability wave that travels along the bubble interface down towards the supply surface, (D) departure of the bubble from the supply surface, and (E) extinction of the bubble, which occurs when the bubble is completely extracted. Examples of Type 2 and Type 3 bubbles are shown in Figure 3b.

The focus of the experimental component of the present study is the departure characteristics of Type 1 bubbles. Of interest are the times, based on the time stamps on the movies, that correspond to each of the five events. Also provided are the effective diameter as a function of time, as determined from image processing of the high-speed movies, and the diameters at both rupture and departure for each bubble studied.

Figure 4a shows the effective bubble diameter, hereafter referred to simply as bubble diameter, as a function of time for eight representative bubbles acquired for the diabatic condition where the gap height is equal to 0.75 mm. Shown on the figure are the points of rupture, departure, and extinction for each bubble. Evident from Figure 4a is that the trends are very repeatable with a small degree of scatter. The averaged rupture diameter and averaged departure diameter for this case are 0.72 mm and 0.58 mm, respectively. The averaged departure frequency of the bubbles is 161 Hz. These experimental values, along with those for all diabatic and adiabatic test conditions and the number of bubbles used in their assessment, are tabulated in Table 1.

The maximum uncertainties in experimental data reported in Table 1 are ±0.02 mm for diameter and gap height, and ±3 Hz for frequency.

The temporal variations in diameter for ten different bubbles for the adiabatic case at a gap height of 1.22 mm are shown
Figure 4 Experimental effective bubble diameter versus time with inception (I), rupture (R), departure (D), and extinction (E) points identified for each bubble for (a) diabatic case for $H = 0.75$ mm and 8 bubbles, and (b) adiabatic case for $H = 1.22$ mm and 10 bubbles.

Figure 4b. Again, the trends are very consistent with a small degree of scatter. However, there appears to be more of an oscillation in the diameter between the rupture and departure events than in the diabatic case. Although the cause for this oscillation is unknown, it may be due to a time varying extraction diameter. Unfortunately, measuring a diameter at an interface from the images is difficult, as noted by Cooper et al. [20].

The averaged experimental diameter data in Table 1 are plotted in Figure 5 as a function of gap height. There is a decrease in both the rupture and departure diameters as the gap height is decreased. Linear regressions were performed on the diabatic and adiabatic rupture diameters and departure diameters as a function of gap height, with both in units of millimeters. The standard error of each fit is provided in parentheses following the equation. For the diabatic bubble rupture diameter, the equation is

$$D_R = 0.83H + 0.16 \pm 0.08 \text{ mm}$$  (1)

For the adiabatic case, the equation is

$$D_R = 0.62H + 0.84 \pm 0.15 \text{ mm}$$  (2)

For the diabatic bubble departure diameter, the equation is

$$D_D = 0.81H \pm 0.03 \text{ mm}$$  (3)

For the adiabatic bubble departure diameter, the equation is

$$D_D = 0.84H \pm 0.14 \text{ mm}$$  (4)

Recall that these correlations are only valid for the range of $H$ used in the fit; therefore, they are not expected to predict unconfined cases for departure or situations in which the gap height approaches zero. Diameter results estimated from these equations are provided in Table 1, and denoted as correlation values.

Experimental bubble departure frequency for both diabatic and adiabatic cases are also provided in Figure 5. There is an observed increase in departure frequency with decreases in gap height. The dimensionless frequency, reported in the form of the
Table 1  Averaged bubble rupture diameters, departure diameters and departure frequencies from experiments, correlations, bubble model and/or static force balance.

<table>
<thead>
<tr>
<th>Number of bubbles</th>
<th>Diabatic: $H$, mm</th>
<th>Adiabatic: $H$, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.52</td>
<td>0.75</td>
</tr>
<tr>
<td>$D_R$ (mm)</td>
<td>Experiments</td>
<td>0.56</td>
</tr>
<tr>
<td>Rupture correlation: Eq. (1)</td>
<td>0.59</td>
<td>0.78</td>
</tr>
<tr>
<td>$D_D$ (mm)</td>
<td>Experiments</td>
<td>0.42</td>
</tr>
<tr>
<td>Departure correlation: Eq. (3)</td>
<td>0.42</td>
<td>0.61</td>
</tr>
<tr>
<td>Force balance</td>
<td>Experiments</td>
<td>0.44</td>
</tr>
<tr>
<td>$f_D$ (Hz)</td>
<td>Experiments</td>
<td>260</td>
</tr>
<tr>
<td>Strouhal correlation: Eq. (7)</td>
<td>263</td>
<td>155</td>
</tr>
<tr>
<td>Diameter model</td>
<td>190</td>
<td>162</td>
</tr>
</tbody>
</table>

Note. Correlations include those for the bubble departure diameter and frequency. Included is the number of bubbles used in computing the averages.

Strouhal number,

$$St = f_D \sqrt{H/g}$$  \tag{5}

is presented as a function of dimensionless gap height in Figure 6. A characteristic velocity similar to that proposed in Haberman and Morton [21],

$$u \propto \sqrt{g D_D} \approx \sqrt{g H}$$  \tag{6}

is used in defining the Strouhal number. According to Eqs. (3) and (4), $H$ is linearly proportional to $D_D$; therefore, $H$ is used as the characteristic dimension in the velocity term. The gap height in Figure 6 is normalized by the unconfined bubble departure diameter, $D_{UD}$. To be of practical utility, $D_{UD}$ is computed using the correlation given by Kocamustafaogullari [13] for the diabatic cases. For the adiabatic case, $D_{UD}$ is determined using a static force balance from Thornicroft et al. [14] that includes growth, buoyancy and surface tension forces and assumes a 90-degree contact angle.

The lines in Figure 6 represent linear regressions of the data. For the diabatic case, with the standard error of the fit provided in parentheses, the regression equation is

$$St = 2.0 \left( \frac{H}{D_{UD}} \right)^{1/2} - 1.5 \pm 0.07$$  \tag{7}

and the equation is

$$St = 0.41 \left( \frac{H}{D_{UD}} \right)^{1/2} - 0.27 \pm 0.03$$  \tag{8}

for the adiabatic case. Evident from these relations and the definition of $St$ from Eq. (5) is that the gap height and bubble departure diameters are proportionally related. Recall that according to Eqs. (3) and (4) the departure diameter is proportional to the gap height; therefore, the departure frequency is inversely related to the departure diameter. This trend is consistent with that for unconfined bubbles, as is reported in Carey [22]. Again, frequency values using these equations are pro-

Figure 6  Strouhal number as a function of dimensionless gap height for diabatic and adiabatic cases. Linear regressions are also provided: Eq. (7) for diabatic case and Eq. (8) for adiabatic case.
vided in Table 1 for comparison and are denoted as correlation values.

The data were also used to determine the time intervals between events, which are shown in Figure 7. For both the diabatic and adiabatic cases, the times from inception to contact and from inception to departure decrease dramatically as the gap height is reduced. However, there is no discernable trend with gap height in the time between contact and rupture. The time it takes from departure to extinction also decreases with decreases in gap height. This decrease may be related to the smaller volumes that need to be extracted for smaller bubbles. However, the predictability of this time would also depend upon the rate of extraction, which depends upon the extraction area, which may also vary with gap height. The time between rupture and departure for the diabatic cases decreases with gap height from 3 to 0.8 ms, which from Figure 7 appears small when compared to the decrease in the total duration of the bubble. There is no discernable trend in this time interval for the adiabatic case. As expected for a constant volumetric injection rate, as is the case for the adiabatic case, the wait time is insignificant. The wait time is also negligible for the diabatic case, which suggests that the volumetric growth rate of the bubble is also constant for diabatic conditions.

The bubble volume variation with time for a single bubble is shown in Figure 8a for the diabatic case for each of the gap heights of 0.52, 0.75 and 1.07 mm and in Figure 8b for the adiabatic case for a 2.14 mm gap height. The points of departure (D), rupture (R), and extinction (E) are specific to these particular bubbles and are presented because they represent the boundaries defining the various regimes. Starting with the region between departure and extinction, the slopes are fairly linear up until very close to extinction. Cappello et al. [10] found Darcy's Law, as reported from DeWeist [23],

$$\dot{V}_{ext} = \frac{K \Delta P A_{ext}}{\mu g \delta}$$

(9)

to predict well the extraction process when the area of the bubble in contact with the extraction surface is used. Because the slopes of the extraction only regime vary between the three diabatic cases presented, whereas the membrane characteristics, driving
pressure difference, and thermophysical properties do not, this suggests that the extraction area likely depends upon gap height. It appears the extraction areas decrease with gap height, which is consistent with the smaller bubble size at smaller $H$.

In Figure 8a, between the point of rupture and point of departure for the gap heights of 0.75 and 1.07 mm, the slope also appears to be fairly linear. This suggests that the extraction and growth rates are likely both constant throughout. It could also be that the growth rate decreases as the extraction rate increases, although what would be contributing to a decrease in growth is unclear. For the smallest gap height of 0.52 mm for the diabatic case in Figure 8a and for the 2.14 mm adiabatic case in Figure 8b, however, there is clearly a change in slope from near rupture to near departure. For the adiabatic case with a constant growth rate, this change indicates that the rate of extraction changes between these two points. This is also assumed for the diabatic case. The only variable that could be responsible for the change in extraction rate, according to Darcy’s Law, is the extraction area. Unfortunately, it is difficult to tell what is happening without a study of the three-phase contact-line dynamics at the extraction surface.

The injection rate is well approximated by the slope of the curve between inception (I) and rupture (R) in Figure 8b. From Figure 8a it is evident that the volumetric growth rate from inception to near rupture can also be fairly well approximated as constant. However, the fact that the slopes decrease with decreasing $H$ for these three diabatic test cases although the heat supplied is constant is noteworthy. For a fixed heat input to a bubble, in which $C'$ is some fraction of heat, $q$, supplied to the surface that enters the bubble, a constant volumetric growth rate can be predicted from

$$
\dot{V}_G = \frac{C'q}{\rho_e h_{ic}}
$$

(10)

For an unconfined bubble $C'$ was empirically determined to be 0.06 for the measured volumetric growth rate of 41.7 mm$^3$/s.

The observed differences in volumetric growth rate during the growth-only regime suggest that the heat input to the bubble, that is, the coefficient $C'$, is likely influenced by the presence and height of the gap.

To help explain what may contribute to this change in volumetric flow rate, a plot of experimental volumetric flow rate versus gap height for the diabatic cases is provided in Figure 9. A quadratic regression analysis, in which $H$ is in millimeters, yields

$$
\dot{V}_G = -70H^2 + 184H - 49.7 (\pm 8.8 \text{ mm}^3/\text{s})
$$

(11)

Because the correlations developed in the previous section are limited in applicability to conditions in which the data were acquired, a theoretical model to predict the diameter and frequency at which bubbles depart is preferred. To determine the departure criterion for unconfined bubble departure, Thorncroft et al. [14] started with the transient momentum equation valid for bubble formation in a pool, the equation of which is provided.

**Bubble Diameter Model and Force Balance**

The experimental volumetric flow rate values are also tabulated in Table 2 along with the total number of bubbles recorded, the number of bubbles experiencing rupture prior to departure (Type 1), the number of bubbles that coalesce while still attached to the supply surface (Type 2), and the number of single bubbles that depart prior to rupture (Type 3). See Figure 3b for examples of these types of bubbles. Also note that the number of bubbles may differ from those in Table 1, which has the number of bubbles included in assessing the averages reported. Not all bubbles were deemed adequate for diameter and/or frequency assessments.

Apparent from Figure 9 is that for a gap height of 0.75 mm the experimental volumetric flow rate of 41.7 mm$^3$/s for unconfined bubbles is a good approximation. Note that this gap height is approximately half of the unconfined bubble departure diameter, $D_{UD}$. For the lowest gap height of 0.52 mm, the heat input to the bubble appears to be suppressed as indicated by a lower volumetric growth rate. Although a decrease in heat input is undesirable, the added benefit of being able to more closely space nucleation sites may counteract this drawback. For gap heights greater than 1 mm, the heat input to the bubble appears to be improved over that of the unconfined case. Although the reason for this is not clear, evident from Table 2 is that the occurrence of coalesced bubbles and bubbles that depart prior to rupture increases significantly, from essentially zero to over 50%. It is believed that the hydrodynamics may be altering the amount of supplied heat that gets into a Type 1 bubble following one of the other types of bubbles.
Table 2: Experimental volumetric growth rate for Type 1 bubbles, the number of bubbles for each bubble type, and the number of bubbles observed for each gap height for the diabatic case.

<table>
<thead>
<tr>
<th>H (mm)</th>
<th>All bubbles</th>
<th>Type 1</th>
<th>Type 2</th>
<th>Type 3</th>
<th>˙VG (mm³/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.52</td>
<td>112</td>
<td>112</td>
<td>0</td>
<td>0</td>
<td>31</td>
</tr>
<tr>
<td>0.75</td>
<td>156</td>
<td>153</td>
<td>3</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>1.07</td>
<td>177</td>
<td>79</td>
<td>85</td>
<td>13</td>
<td>77</td>
</tr>
<tr>
<td>1.40</td>
<td>247</td>
<td>85</td>
<td>141</td>
<td>21</td>
<td>78</td>
</tr>
<tr>
<td>1.60</td>
<td>220</td>
<td>47</td>
<td>148</td>
<td>25</td>
<td>65</td>
</tr>
<tr>
<td>1.67</td>
<td>224</td>
<td>40</td>
<td>160</td>
<td>24</td>
<td>52</td>
</tr>
<tr>
<td>1.90</td>
<td>277</td>
<td>9</td>
<td>225</td>
<td>43</td>
<td>55</td>
</tr>
</tbody>
</table>

Note. Types of bubble are identified in Figure 3.

\[
\frac{d (\mu u)}{dt} = F_B + F_G + F_{\sigma, sup} + R_{y, sup} + (\bar{\mu}u)_i \quad (12)
\]

with the inlet momentum included and \(F_B\) representing the net buoyant force. The momentum balance reduces to a static force balance at departure because the transient term is assumed negligible due to the small bubble mass and the inlet momentum also goes to zero as the bubble is pinched off. Also, the reaction force, \(R_{y, sup}\), that results from van der Waal forces acting through the meniscus at the supply surface goes to zero at departure [14].

The momentum balance in Eq. (12) modified for the conditions with confined extraction is

\[
\frac{d (\mu u)}{dt} = F_B + F_G + F_{\sigma, sup} + R_{y, sup} + R_{y, ext} + (\bar{\mu}u)_i - (\bar{\mu}u)_e \quad (13)
\]

For confined extraction the reaction force at the extraction surface, \(R_{y, ext}\), is also likely to be negligible, not because of lift-off, which does not occur at this surface at departure, but because of the porous hydrophobic nature at that surface. Unlike the inlet momentum, however, the exit momentum does not approach zero at bubble departure. The magnitudes of the terms in Eq. (13), except for the reaction forces and transient momentum term, are computed over the course of the bubble lifetime in order to elucidate relevant physics and determine the applicability of applying a static force balance to predict departure diameter, that is, the practicality of neglecting the exit momentum.

If the diameter of the bubble is known as a function of time, as is represented schematically in Figure 10a, the predicted departure diameter can be used in conjunction with the diameter versus time relation to determine the time it takes between inception and departure. Given that the wait time is negligible, as noted from Figure 7, the inverse of this time would provide a prediction of the bubble departure frequency.

Because the volumetric flow rate during the growth only regime can be approximated as constant, as shown schematically in Figure 10b as a linear line on a bubble volume versus time plot, and because extraction can be modeled through Darcy’s Law, there exists the potential to develop a theoretical model that predicts bubble diameter as a function of time. The theoretical bubble diameter model is based on conservation of mass for the three different regimes shown in Figure 11. Also shown in Figure 11 are the force terms and momentum terms to be assessed during the lifetime of the bubble. Regime I, shown in Figure 11a, only has mass entering and is known as the growth only regime, starting at inception and ending at the point the bubble ruptures at the extraction surface. To be considered in the force/momentum analysis are the inlet momentum, net buoyant force, surface tension force at the supply surface, and the growth force. Regime II experiences both growth and extraction, starting at the point of rupture and ending at the point at which the bubble departs from the supply surface. Figure 11b shows the force and momentum terms to be considered in Regime II, which...
includes the net buoyancy force, the surface tension force at the 
supply surface, and the inlet momentum from Regime I, plus 
the exit momentum and surface tension at the extraction surface. 
The vertical growth rate upon rupture is negligible; hence, the 
growth force in this regime is considered negligible. Regime III, 
initiated at departure and ending at extinction, is shown in Figure 
11c. Although not important to model in terms of finding the de-
parture diameter and departure frequency, including Regime III 
can aid in the understanding of the physics. In Regime III, only 
exit momentum and the surface tension force at the extraction 
surface are considered.

**Bubble Diameter Model**

The control volume used in the development of the bubble 
diameter model is shown in Figure 1. In all cases the density of 
the bubble is assumed constant.

**Regime I.** A transient mass balance in which mass enters the 
system at a fixed rate equal to $\dot{V}_G$ yields

$$D_I(t) = \frac{6\sqrt{6}t}{\pi}$$

where time, $t$, is set to zero at bubble inception. This equation 
is valid for all of Regime I, from inception (I) to rupture (R), 
where $t_R$ is determined by dividing the volume at rupture by 
the growth rate, $\dot{V}_G$. The volume at rupture is determined using 
rupture diameters predicted from Eqs. (1) and (2) for diabatic 
and adiabatic cases, respectively.

**Regime II.** After the bubble ruptures, growth is accompa-
nied by extraction. For the present study, it is assumed that the 
extraction process does not influence the rate of growth and 
that the rate at which mass leaves by extraction is consistent 
with Darcy's Law, provided in Eq. (9). A transient mass balance 
considering mass entering and leaving the system yields

$$D_{II}(t) = \frac{6}{\pi} \left[ V_R + \dot{V}_G (t - t_R) - \frac{K A_{ext,II}}{\mu g \delta} (t - t_R) \right]$$

This equation is only valid from rupture (R) to departure (D), 
that is, $t_R < t \leq t_D$. This equation is also only valid for a constant 
extraction area, when in reality the extraction area is expected 
to increase from a value of zero at rupture. A modified form of 
this equation is presented after a variation in extraction area is 
assumed, at which point the prediction of $t_D$ is also addressed.

**Regime III.** Following bubble departure, the rate of growth 
goes to zero and the mass inside the bubble is balanced by 
the rate of mass removed. Again, using Darcy’s Law to model 
extraction yields

$$D_{III}(t) = \frac{6}{\pi} \left[ V_D - \frac{K A_{ext,III}}{\mu g \delta} (t - t_D) \right]$$

which is applicable from departure (D) to extinction (E). The ex-
traction area, $A_{ext,III}$, is assumed constant because of the constant 
extraction rates observed during Regime III.

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**Figure 11** Force and momentum terms considered on a bubble during the 
regimes: (a) Regime I: growth only, (b) Regime II: growth with extraction, and 
(c) Regime III: extraction only.
Relevant information needed for Eqs. (14) through (16) are the following: (i) growth rate, (ii) diameter (volume) of the bubble at rupture, which is used to indicate the onset of Regime II, (iii) the diameter of the extraction area during Regime II, which is expected to vary over time and start at a value of zero at the point of rupture, (iv) the bubble departure diameter (volume), used to indicate when to turn off growth at the onset of Regime III, and (v) the diameter of the extraction area during Regime III.

The growth rate for the diabatic case is simply the volumetric injection rate of the air, the averaged value of which is 88.1 mm$^3$/s. For these initial stages of model development, the correlation provided by Eq. (11) is used for the diabatic case. Ideally, a theoretical model to predict the heat input to the bubble would be available for use.

The diameter of the bubble at which rupture occurs, $D_R$, may eventually also be theoretically predicted, perhaps using surface energy. However, at these initial stages of model development, the empirical values predicted from Eq. (1) for the diabatic case and Eq. (2) for the adiabatic case are used to assess the rupture volume.

The departure diameter is generally predicted as the bubble diameter that balances the static forces immediately prior to departure. The forces to employ in a static force balance for confined extraction are assessed in the next section.

The diameter of the bubble in contact with the extraction surface is needed to compute the extraction area in Darcy’s Law and the surface tension at the extraction surface. For the present purposes the extraction rates in Regime III are fairly constant and the surface tension at the extraction surface, $\sigma$, is assumed that

$$F_B = \frac{\pi}{6} \left( \rho_l - \rho_g \right) g D^3$$

respectively, where $\dot{r}$ is determined from

$$\dot{r} = kn \tau^{-1}$$

For both adiabatic and diabatic cases, $n = 1/3$. For the diabatic case, $k$ in Eq. (24) is 0.00277 m/s$^{1/3}$ and $d_{sup}$ is simply the orifice diameter. For the diabatic case, $k$ is 0.00215 m/s$^{1/3}$ and

$$d_{sup} = D \sin \theta_{sup}$$

In Regimes II and III, the exit momentum is

$$\dot{m} \Delta \frac{1}{\rho} \left( \frac{K \Delta P}{\mu_s b} \right)^2$$

which acts upward, as does the surface tension force at the extraction surface,

$$F_{s,ext} = d_{ext} \sigma \pi \sin \theta_{ext}$$

In summary, Eqs. (14), (16), (17), and (19) are used to assess bubble diameter versus time.

**Force/Momentum Assessment**

In Regimes I and II, inlet momentum and net buoyancy force are computed using volumetric growth rates and bubble diameters predicted from the bubble diameter model. Both act upward. The inlet momentum is computed from

$$\left( \dot{m} \Delta \right) = \frac{4 \rho_l V_G^2}{\pi d_{sup}^3}$$

and the net buoyancy is found using

$$F_B = \frac{\pi}{6} \left( \rho_l - \rho_g \right) g D^3$$

The surface tension force at the supply surface and the growth force from [14], both of which act downward, are

$$F_{s,Sup} = -d_{sup} \sigma \pi \sin \theta_{sup}$$

and

$$F_G = -\frac{\pi}{4} \rho_l D^2 \left( 1.64 \dot{r}^2 \right)$$

For both adiabatic and diabatic cases, $n = 1/3$. For the diabatic case, $k$ in Eq. (24) is 0.00277 m/s$^{1/3}$ and $d_{sup}$ is simply the orifice diameter. For the diabatic case, $k$ is 0.00215 m/s$^{1/3}$ and

$$d_{sup} = D \sin \theta_{sup}$$

In Regimes II and III, the exit momentum is

$$\dot{m} \Delta \frac{1}{\rho} \left( \frac{K \Delta P}{\mu_s b} \right)^2$$

which acts upward, as does the surface tension force at the extraction surface,

$$F_{s,ext} = d_{ext} \sigma \pi \sin \theta_{ext}$$

The extraction area and extraction diameter for the appropriate regime, that is, II or III, should be employed. Each force and momentum term is computed for each point in time using the bubble diameter determined from the bubble diameter model.

Equation (24) was derived for a spherical bubble growing normal to the supply surface; therefore, it should become zero when the bubble first makes contact with the extraction surface. However, bubble growth may not be spherical and a bubble may
make contact at a diameter smaller than the gap height. This is evident from Figure 5, which shows that the rupture diameter can be less than the gap height. Therefore, the growth force in the current model is terminated at rupture.

**Bubble Model Validation and Preliminary Model Results**

To validate the bubble diameter model, departure diameters and departure frequencies for both diabatically and adiatically generated bubbles under unconfined conditions were predicted. A static force balance including growth and net buoyant and surface tension forces was employed to assess departure diameters. Based on this departure diameter, the bubble diameter model was used to predict diameter versus time during the growth phase until the departure diameter condition is reached. This resulted in a prediction of the time from inception to departure. By taking the inverse of this time, the departure frequency was determined. Thermophysical properties for the diabatic study were determined for saturated conditions at 1 atm, and at 20°C and 1 atm for the adiabatic study.

Once the static force balance and bubble diameter model are validated, bubble diameters, forces, and momentum terms can be predicted for each point in time under confined extraction conditions. Ideally, various combinations of $\theta_{\text{ext}}$ and $\theta_{\text{sup}}$ can be assessed to find the single combination yielding the best prediction of the experimental departure diameters and frequencies, as well as the trends in the three regimes. Experimentally characterized values of permeability, $K$, are $6.2 \times 10^{-15}$ m² and $7.5 \times 10^{-15}$ m², respectively, for vapor and dry air and are used in the current model (Cappello et al. [10]).

**Model Validation**

For the diabatic case, the unconfined departure diameter and departure frequency were predicted using the unconfined bubble growth rate of 41.7 mm³/s and a supply surface contact angle of 14.5 degrees. The predicted values are 1.54 mm and 21.8 Hz, compared with experimental values of 1.53 mm and 22.4 Hz, respectively, which is used as validation of the model. Note that the contact angle, although on the low end, is within the range reported for static contact angles on a silicon surface according to Williams and Goodman [24].

For the adiabatic case, the unconfined departure diameter and departure frequency were predicted using a bubble growth rate of 88.1 mm³/s and a 90-degree contact angle at the supply surface. Predicted values are 2.85 mm and 7.24 Hz, compared with 2.88 mm and 7.2 Hz determined experimentally.

**Bubble Departure Diameters from Static Force Balance**

Plotted in Figure 12 are the time-varying forces and the inlet and exit momentums shown in Figure 11, as well as the sum of only the forces, $\Sigma$, for the diabatic case with a gap height of 0.75 mm. In this particular case, and all others under the current test conditions, both inlet and exit momentum terms are negligible (two orders of magnitude smaller than forces, hence not clearly distinguishable).

In terms of the physics, it is evident from Figure 12 that the surface tension at the extraction surface, the magnitude of which is affected by the diameter and contact angle at the three-phase contact line, is the dominant mechanism for Type 1 bubble departure. This finding suggests that the proposed configuration with small gap heights can be used in a variety of orientations and in low gravity environments.

Also evident from Figure 12 is that the growth force is significant only immediately following inception. However, because the vertical increase in growth, $r$, is restricted by the extraction surface, the growth force is only important until the bubble reaches the contact surface. Because the exact time at which contact occurs is difficult to determine, and therefore the diameter at this time, for consistency the growth force is set to zero at the point of rupture. Therefore, for Type 1 bubbles undergoing confined extraction, the departure diameter is predicted from a static force balance in which the buoyant force and the surface tension forces at the supply surface and the extraction surface are included.

**Bubble Departure Frequencies from Bubble Model**

Diabatic volumetric growth rates from Eq. (11) were used in the bubble diameter model with confined extraction. Initially
the 14.5-degree contact angle used in validating the model was employed with a 125-degree static contact angle measured at the hydrophobic surface using a sessile drop. However, the results did not compare well with experimental data. Therefore, the values of $\theta_{\text{ext}}$ and $\theta_{\text{sup}}$ that best fit the trends over the range of test conditions were 165 degrees and 30 degrees, respectively. Although the value used at the supply surface is greater than that found for matching the data for unconfined bubble departure, it is still in the range of values reported for static contact angles on a silicon surface [24]. Furthermore, given the additional upward force acting on the bubble after rupture, the supply surface contact angle might be expected to be greater than for the unconfined case.

Likewise, although the static contact angle of a sessile water droplet on the hydrophobic extraction surface in the presence of air was found to be 125 degrees, the existence of downward gravitational forces, the fact that it is a vapor/liquid interface as opposed to a gas/liquid interface in contact with the hydrophobic membrane, and the fact that vapor is being removed could all contribute to a higher contact angle at the extraction surface. In any event, the results are highly sensitive to the contact angles at both surfaces. Ideally, a theoretical model for predicting dynamic contact angles at both surfaces would be quite beneficial.

For the diabatic case with a gap height of 0.75 mm, a departure diameter of 0.64 mm was predicted using the static force balance, which is within 10% of the averaged experimental value. The diameter of the bubble as a function of time was predicted using the bubble diameter model, the results of which are provided in Figure 13a, with data from Figure 4a superimposed. The point of rupture, R, is predicted from Eq. (1). The point of departure, D, is predicted using departure diameter determined from a static force balance. The inverse of the time from inception to when the departure diameter is reached is used to assess the departure frequency. It was predicted to be 162 Hz, which is within 1% of the experimental value of 161 Hz. The bubble diameters predicted clearly represent the experimental trends and departure conditions fairly well for this case. The departure diameter and frequency predictions for all diabatic cases are tabulated in Table 1. Departure diameters predicted from a static force balance are within 10% of experimental values and departure frequencies predicted from the bubble diameter model are within 30% of experimental values.

The agreement in trends is not quite as good for the diabatic case with a gap height of 0.52 mm as it was for the 0.75-mm case, as is evident from Figure 13b. However, because of the use of empirical estimates for the volumetric growth rate and for the rupture diameter, the assumptions regarding unknown trends in $d_{\text{ext,II}}$ and $d_{\text{ext,III}}$, and finally the sensitivity to contact angles, the predictive nature of the bubble diameter model coupled with the static force balance is considered to be very good and with great potential once the empiricism is minimized or removed. For the diabatic case, the value of $\theta_{\text{sup}}$ can be assumed to be essentially constant [14], whereas $d_{\text{sup}}$ varies with bubble diameter, as noted from Eq. (25). Recall also that Eq. (3) suggests a decrease in bubble diameter with a decrease in gap height. Therefore, for the diabatic cases, in accordance with Eq. (22) for a fixed $\theta_{\text{sup}}$, a decrease in gap height yields a decrease in the only downward acting force, that is, the surface tension force at the supply surface. This decrease in the only downward acting force is consistent with a prediction of smaller departure diameters from a static force balance, as expected for smaller gap heights.

![Figure 13](image_url) Predicted bubble diameter versus time for diabatic case overlaid experimental data for (a) $H = 0.75$ mm and (b) $H = 0.52$ mm. Points R and D represent $D_R$ from Eq. (1) and $D_D$ from a static force balance.
For the adiabetic case, however, \( d_{\text{sup}} \) is constant and equal to the supply orifice diameter. Therefore, to force a single combination of \( \theta_{\text{sup}} \) and \( \theta_{\text{ext}} \) for all three adiabetic cases would result in a single value for the downward force that is independent of gap height. The consequence of this is that the static force balance yields a constant bubble departure diameter, independent of the gap height. Because this is not the case from observations, a relation for the dependence of \( \theta_{\text{sup}} \) on bubble diameter is needed.

Therefore, the value of \( D_2 \) for the adiabetic cases is determined using Eq. (4), the empirical result of departure diameter versus gap height, rather than from a static force balance. This value of \( D_2 \) is then used with the bubble diameter model to predict the bubble departure frequency. In Figure 14, the adiabetic data from Figure 4b are again plotted with the bubble diameter predictions, with rupture diameter and departure diameter predictions overlaid. The trend in diameter is well predicted, although the departure diameter is somewhat higher, as noted from Table 1, than the experimental value. It is expected that by including a larger data set the experimental averages will result in a better predictive correlations for both rupture and departure diameters. The departure frequencies predicted using the bubble departure model reported in Table 1 are within 10% to 30% of the experiment values.

In summary, a static force balance that includes the buoyancy force, and the surface tension forces at both the supply and extraction surface, predicts fairly well diabatic bubble departure diameters. The semiempirical bubble diameter model yields fairly decent values of bubble departure frequency for both diabatic and adiabetic cases. However, both the bubble diameter model and the static force balance would benefit from having more theoretically based predictions of the three-phase contact line dynamics at the extraction surface and conditions for bubble rupture. The force balance could be greatly improved with a better understanding of, and a better ability to predict, the dynamic contact angles at the hydrophobic porous surface while undergoing extraction and at the supply surface for the adiabetic case.

Finally, there is obviously a practical limitation to this technique. If the extraction rate is less than the rate at which vapor is generated for cooling applications, there will be a vapor layer that could easily lead to dryout. For a given volumetric growth rate, which is related to the heat input by Eq. (10), the following can be used to assess the minimum acceptable gap height, \( H \):

\[
H \geq \sqrt{\frac{4V_c\mu_p\delta}{\pi B^2K\Delta P}}
\]

In this equation \( B \) is the slope of the departure diameter correlation in Eq. (3), and \( \delta, K, \mu_p, \) and \( \Delta P \) are characteristics of the extraction conditions determined by the application.

**CONCLUSIONS AND RECOMMENDATIONS**

High-speed movies of confined extraction of individual vapor and gas bubbles supplied at a constant volumetric flow rate were studied for a range of confinement, or gap, heights. Of the types of bubbles identified, the ones studied are those that ruptured prior to departing from the surface from which they were supplied. These are most relevant to addressing spatiotemporal cooling needs. The bubble dynamics are categorized into three regimes: Regime I: bubble growth only, Regime II: simultaneous growth with extraction, which follows rupture of the bubble at the extraction surface; and Regime III: bubble extraction only, which occurs after the bubble departs from the supply surface.

Correlations for predicting the diameter at which the bubbles rupture and at which they depart and for predicting the departure frequency were generated for both the diabatic and adiabetic cases. The bubbles depart at a fixed fraction of the gap height whereas the departure frequency is inversely related to the gap height. This inverse relation between diameter and frequency is typical of bubble departure under unconfined conditions. Both the diabatic and adiabetic cases experienced no wait times.

At these initial stages of model development, empirical correlations were used in a static force balance to determine bubble departure diameter for a given condition, which was subsequently used in a theoretically based dynamic bubble diameter model for predicting bubble departure frequencies. A study of the forces acting on the bubble as a function of time shows that the surface tension force at the extraction surface is the driving factor for inducing small bubble departure diameters. Other important forces to include are the surface tension at the supply surface and net buoyancy. The exit momentum and inlet momentum are negligible. Also, the presence of the confinement surface suppresses the growth force after the bubble makes contact with this surface. Because the time of contact is hard to...
predict, the growth force is turned off after rupture and, unlike in the static force balance for an unconfined situation, is not included in a static force balance for confined extraction. The force balance is highly sensitive to the contact angle at both the extraction surface and the supply surface. A practical limitation on the minimum gap height for a given bubble injection rate is provided, where the injection rate can theoretically be determined from the heat input.

To improve the bubble diameter model, an experimental relation as a minimum, or preferably a theoretically based relation, is needed for the time varying extraction area following rupture. A study of the mechanism(s) responsible for bubble rupture and a study of the dynamic contact angle at a hydrophobic porous surface during the extraction process are also of interest. In addition, a study as to what influence the gap height plays on the fraction of supplied heat that enters the bubble would be beneficial.

**NOMENCLATURE**

850 $A$ area m$^2$

855 $B$ Constant in Eq. (28)—

860 $C'$ fraction of heat entering bubble in Eq. (10)—

865 $D$ diameter of three-dimensional objects: bubbles m

870 $d$ diameter of two-dimensional objects: contact areas m

875 $f$ frequency Hz

$F$ force N

$g$ gravitational acceleration kg-m/s$^2$

$h_{lv}$ enthalpy of phase change J/kg

$H$ extraction surface gap height m

$k$ growth rate constant in Eq. (24) m/s$^{1/3}$

$m$ mass flow rate kg/s

$\Delta P$ power input N/m$^2$

$q$ time and space averaged heat flux W

$\dot{r}$ radial growth rate m/s

880 $R_e$ reaction force normal to supply/extraction surfaces N

$t$ time s

$St$ Strouhal number —

$u$ velocity m/s

$V$ volume m$^3$

885 $\dot{V}$ volumetric flow rate m$^3$/s

**Greek Symbols**

890 $\theta$ contact angle degrees

$\delta$ membrane thickness m

$\sigma$ surface tension N/m

905 $\rho$ density kg/m$^3$

$K$ membrane permeability m$^2$

$\mu$ dynamic viscosity N-s/m$^2$

$\Sigma$ summation of forces N

**Subscripts**

895 B net buoyant

D departure

e exit

ext extraction

g gas for adiabatic and vapor for diabatic studies

G growth

i inlet

l liquid phase

R rupture

sup supply

UD unconfined departure

$\sigma$ surface tension

I Regime I

II Regime II

III Regime III

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