Modeling criteria for extraction regime transitions for microscale in-situ vapor extraction applications

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A major detriment of two-phase microscale flow systems is a significantly large pressure drop. For flow boiling the potential for flow instabilities is also a major concern. Both disadvantages may be suppressed by extracting vapor through a hydrophobic porous wall of the channel as a means to reduce the channel vapor fraction. The vapor extraction may occur as different regimes either as evaporation, bubble extraction or a combination of both. In the design of vapor extraction systems, it is important to accurately predict extraction rates, different extraction regimes, and the effect of extraction on the heat transfer and channel flow conditions. This study focuses on two parts: the development of physic-based models for the transition criteria among (i) the extraction mechanism regimes, and (ii) the extraction flow regimes for microscale flow boiling. The identification and conditions for the various extraction regimes are discussed and criteria for transition are developed based on physical concepts. Six potential extraction mechanism regimes are identified: (a) no extraction, (b) evaporation, (c) bubble extraction, (d) bubble extraction with partial liquid blockage, (e) bubble extraction with evaporation, and (f) liquid break-through. Based on the criteria for the extraction mechanism regimes, the rate of vapor extraction is modeled and used to analyze the effects of vapor extraction on the dynamics of two-phase flow boiling. The results show six extraction flow regimes for two-phase flow boiling: (i) single-phase evaporation, (ii) two-phase evaporation – bubble collapse, (iii) full extraction – stable, (iv) full extraction – unstable, (v) partial extraction – stable and (vi) partial extraction – unstable.

1. Introduction

Due to the increased development of high performance electronic devices in the last few decades, as well as their miniaturization, the large increase in heat dissipation per unit volume has become a major concern. Cooling technologies utilizing two-phase microscale heat sinks have advantages due to high surface to volume ratio and large heat transfer coefficients. For two-phase flow boiling, an additional advantage is the relatively small streamwise temperature variation compared with single-phase methods. However, a major issue of flow boiling in microchannels is a correspondingly large pressure drop and accompanying flow instabilities [1,2]. A large pressure drop can lead to large temperature variations along the flow in the two-phase regime. Also, the large pressure drop and thermal oscillations due to flow instabilities may lead to severe mechanical vibration and dry-out [1].

Numerous investigators [2–10] have developed channel modifications as a means to suppress instabilities, which may be categorized into three main groups: (i) use of an inlet restrictor, (ii) application of engineered nucleation sites, and (iii) use of an expanding channel. The first group [2–4] stabilizes flow boiling by placing a flow restrictor at the inlet to decrease the reverse vapor flow. However, the drawback of this method is a large increase in the pressure drop of the system [5]. The second group [5–7] fabricates artificial nucleation sites on the channel walls. This reduces the required superheat surface temperatures and thereby reduces the rapid expansion of the bubble. The third group [8–10] uses an increasing flow cross sectional area along the flow direction causing bubbles to expand downstream rather than upstream, leading to less vapor reverse flow.

An alternative approach using in-situ vapor extraction, which has shown to potential stabilize flow boiling, has been investigated by Salakij et al. [11]. The goal of in-situ vapor extraction is to locally extract the generated vapor from the channel through a hydrophobic porous wall so as to reduce or control the vapor fraction inside the channel. In a thermal management application the extracted vapor would most likely be condensed and reused as a
Several studies suggest that in-situ vapor extraction also has the potential to reduce the system pressure drop while maintaining the benefit of enhanced heat transfer [13–18]. Apreotesi et al. [13,14] experimentally investigated flow boiling of water through a fractal-like branching microchannel network with in-situ vapor extraction, showing a decrease in the system pressure drop with increasing extraction pressure differential. A later work by Salakij et al. [18], using a one dimensional predictive model validated against the experimental results obtain in [13,14], shows up to a 70% decrease in the overall pressure drop. Moreover, the bulk fluid temperature within the channel was shown to decrease indicating the potential of in-situ vapor extraction to decrease the overall operating temperature of the device. David et al. [16] investigated two-phase flow in parallel microchannels with vapor venting, where the flow and venting channels were separated by hydrophobic porous membrane. Their results show a significant decrease in pressure drop. The computational model of the vapor-venting process studied by Fang et al. [17] also confirmed this result showing that vapor-venting helps suppress local dry-out in microchannels.

In order to fully utilize the potential of in-situ vapor extraction for two-phase flow, it is necessary to understand the effects of vapor extraction on heat transfer and flow conditions. The effects of vapor extraction are directly related to the vapor extraction mass flow rate. A number of studies [12–19] have predicted the vapor/gas average velocity, \( V_{\text{extr}} \), transport across the membrane based on Darcy’s law as:

\[
V_{\text{extr}} = \frac{k}{\mu} \nabla P_{\text{extr}}
\]  

(1)

Note that all symbols are identified in the Nomenclature. This model may indeed require added complexities to be accurate for this application. For example, Salakij et al. [12,18] related vacuum membrane distillation to vapor extraction and included evaporation effects on the vapor extraction where the evaporation rate is based on the local vapor pressure gradient across the membrane. Cappello et al. [20] used the dusty gas model, as a general form of Darcy's law, with added effects of membrane compaction to successfully predict gas and superheated vapor transport through the membrane.

Several studies suggest that two-phase hydrodynamic conditions near the membrane affect vapor extraction. Alexander and Wang [21] studied vapor separating from a two-phase microscale flow through a hydrophobic porous plate, called a breather. A...
correlation was developed based on the assumption that the extraction rate is related to the flow velocity as a function of the ratio of the pressure differential across the porous plate to the pressure drop associated with drag on the bubble. Xu et al. [19] studied gas bubble removal in a microscale channel and proposed criteria for complete bubble removal by considering film formation, bubble size, and liquid breakthrough conditions. Cappello et al. [20] studied both air and vapor transport through a porous membrane from two-phase reservoir, showing that the extracted mass flow rate deviates from single-phase transport results. They proposed that these deviations may be caused by the two-phase hydrodynamics near the membrane as well as condensation within the membrane.

The development of both flow regime-based pressure drop and heat transfer models depends strongly on the two-phase flow regime [22]. Flow regime transition criteria for microchannels have been developed using empirical methods, (e.g. [22–26]). For flow boiling with vapor extraction David et al. [27], using flow visualization observed two-phase flow regimes in a vapor-venting microchannel, indicates that stratified flow is a dominated flow regime for low liquid velocities. With increasing liquid velocities, annular flow occurs. The pioneering work by Taitel and Dukler [28], and the later work by Barnea et al. [29], for larger flow channels, developed a dimensionless form of basic relationships to predict the conditions for flow regime transition.

The goal of the present study is to develop a set of physics-based models for the transition criteria among specified regimes applicable to in-situ vapor extraction in two-phase flow in microchannels. Here the flow regimes are identified and the models for extraction are expected to help the development of regime-based extraction mass flow rate predictions, as well as being useful design tools for enhanced performance of liquid–vapor systems.

2. Physics-based regime map

The methodology to develop a physics-based regime map can be divided to four main steps: (i) identify possible regimes and physical conditions for each regime, (ii) develop theoretical models for transition between physical conditions, (iii) nondimensionalize the transition criteria, and (iv) plot transition criteria on an appropriate map. The additional step to make a regime map more practical for a specific application is converting the nondimensional map into a dimensional map for a specific set of conditions, such as a working fluid and flow geometry.

The extraction map is used to identify physical conditions related to vapor extraction. The main goal of this map is to help understand the potential of vapor extraction and provide a tool for design applications. In this study, two main types of extraction maps are proposed: (i) extraction mechanism map, which characterizes how vapor is being extracted, and (ii) extraction flow map, which characterizes the effects of flow boiling on vapor extraction. The former deals with the vapor extraction flow physics through the membrane, while the latter defines the two phase channel flow conditions and how this may affect extraction. Rather than developing a single extraction regime map, that includes all considered extraction regimes, this study focuses on the development of each transition criterion and the corresponding individual map for each specific regime transition.

2.1. Extraction mechanism regimes

Vapor transport can be initiated by applying a pressure differential across a membrane. For a hydrophobic porous membrane, the liquid phase is suppressed from leaking into the membrane pores by surface tension forces, provided the pressure difference across the membrane is below a critical value. This maximum pressure differential across the membrane without liquid leakage is defined as the breakthrough pressure. By applying Young–Laplace equation for a straight capillary pore, the breakthrough pressure is estimated as:

\[ \Delta p_{\text{break}} = \frac{4\sigma_{lv} \cos \theta_{c, mem}}{d_p} \]  

It should be noted that although the breakthrough pressure can be theoretically estimated as in Eq. (2), many researchers experience breakthrough at much lower pressure differential [16, 19, 21, 27]. This may be because of the complex nature of porous membranes having a range of pore sizes and shapes.

Vapor extraction can occur in two modes: bubble extraction and evaporation. Bubble extraction is a consequence of hydrodynamic transport through the membrane, i.e. the vapor phase is extracted through a membrane directly by a pressure differential across the membrane. Evaporation is a result of thermodynamic driven transport where the liquid phase at the membrane evaporates and flows through the membrane via thermal and pressure forces. The evaporation occurs when the vapor pressure of the liquid within the channel in contact with the membrane is greater than the extraction pressure on the opposite side of the membrane. More detail explanation of evaporative transport mechanism can be found in vacuum membrane distillation applications, e.g. [30–33]. Both modes of vapor extraction may coexist depending on extraction conditions and whether liquid, vapor or both phases are in contact with membrane. The physical conditions of vapor extraction and breakthrough are summarized in Table 1.

Extraction mechanism regimes are classified by using membrane transport mechanisms coupled with membrane contact conditions. There are six potential extraction mechanism regimes identified as: (a) No extraction, (b) evaporation, (c) bubble extraction, (d) bubble extraction with partial liquid blockage, (e) bubble extraction with evaporation, and (f) liquid breakthrough. The physical conditions of each regime are summarized in Table 2 where the characteristic of each regime are described as below:

(a) No extraction is when the pressure differential across the membrane is not sufficient to initiate either bubble extraction or evaporation.

(b) Evaporation is when the membrane extraction area is solely in contact with evaporating liquid and the resultant vapor flows through the membrane.

(c) Bubble extraction is when only vapor is in contact with the membrane and the vapor inside the channel is extracted through the membrane.

(d) Bubble extraction with partial liquid blockage is when bubbles are extracted when a fraction of the extraction area is blocked by liquid which does not evaporate.

(e) Bubble extraction with evaporation is similar to (d), however the liquid in contact with the membrane also evaporates, i.e. both modes (b) and (c) occur.

(f) Liquid breakthrough occurs when the pressure difference across the membrane is greater than the breakthrough pressure and liquid flows from the channel through the membrane.

It should be noted that extracting vapor from the channel flow most likely will result in a change in the phenomenological hydrodynamic conditions in the channel. As a result, extraction mechanism regimes can be seen as local conditions along the flow. For example, a high temperature single-phase flow enters a channel, liquid in contact with the membrane may evaporate, where the liquid temperature near the membrane decreases along the flow.
This may suppress downstream evaporation, resulting in a change of regime from pure evaporation to no extraction. Further, a purely stratified flow may change downstream due to extraction into bubbly flow, resulting in a change from purely bubble extraction to bubble extraction with evaporation.

### 2.2. Extraction mechanism regime transition criteria

Extraction mechanism regime transition criteria are based on modeling the transition of the membrane contact phase and membrane transport mechanism. The transition criteria can then be used together with the extraction mechanism regime conditions, summarized in Table 2, to identify the regime. Transition criteria are discussed in detail below.

#### 2.2.1. Membrane contact phase

There are three possible situations for fluid to be in contact with the membrane: (i) vapor only, (ii) liquid only and (iii) a mixture of both phases. Two criteria are used to separate these three situations: liquid film formation and stratification to an intermittent flow regime. For liquid to be the only phase in contact with the membrane during two-phase flow, there must be a liquid film formation on the membrane. To separate between vapor only and mixed phases in contact with the membrane, the contact characteristics are assumed to be related to the two-phase flow regime of the fluid inside the channel. In other words, for stratified flow the vapor phase will generally be the only phase in contact with the membrane (this assumes extraction occurs at the top of the channel). Both phases tend to be in contact with the membrane for an intermittent flow. It should be noted that although stratified flow is rarely found in most microscale flow boiling applications, stratified flow may likely be found in flow boiling with in-situ vapor extraction applications. This is because at least one wall of the channel is formed by a hydrophobic porous membrane, which lacks sufficient surface tension forces to pull the liquid to this wall, causing an extended region of stratified flow. The experiment observations by David et al. [27] support this situation, in that stratified flow is reported to be the dominated flow in low velocity situations with sufficient vapor content.

### 2.2.2. Liquid film formation

Liquid film formation is evaluated based on the fact that the dynamic contact angle decreases with increasing bubble velocity. As the dynamic contact angle approaches zero, the bubble critical velocity necessary to form a liquid film at the surface is estimated based on criteria given by de Gennes et al. [34]:

\[
\begin{align*}
u_{\text{crit}} &= \frac{1}{9\sqrt{3}} \frac{\sigma/\mu}{a} \frac{d}{c_{\text{mem}}} \\
\end{align*}
\]

where \(a\) is a dimensionless parameter estimated to be in the range of 15–20. The liquid film forms when the bubble travels faster than this critical velocity \((u_{\text{bub}} > u_{\text{crit}})\). For simplicity, the bubble velocity may be estimated as the vapor superficial velocity, \(j_v\). In dimensionless form, this film formation criterion is rewritten in terms of a capillary number, \(Ca_v\), as:

\[
\begin{align*}
Ca_v &= \frac{1}{9\sqrt{3}} \frac{\theta_{c,v}}{a} \\
\end{align*}
\]

A liquid film formation map of \(Ca_v\) versus \(\theta_{c,v}\) for the range of values of \(a\) between 15 and 20, based on Eq. (4), is shown in Fig. 1.

A liquid film, once formed, may rupture for a variety of reasons based on the dynamic force balance. However, a simple model can be presented based on the ratio of the film thickness to surface roughness ratio. By assuming annular flow conditions (often found...
in saturated flow boiling in microchannels [35] with a void fraction of $x$ and surface roughness of $\delta_c$, this criterion can be expressed as:

$$x > \left( 1 - \frac{\delta_c}{A_c^{1/2}} \right)^2$$  \hspace{1cm} (5)

The void fraction correlation developed for annular flow by Zivi [36] for a local quality of $x$ can be used:

$$x = \frac{1 + \left( \frac{1 - x}{X} \right) \left( \frac{P_v}{P_l} \right)^{2/3}}{\frac{d}{X}}$$  \hspace{1cm} (6)

By combining Eqs. (5) and (6), the film rupture map given in Fig. 2 shows the film rupture region versus local quality $x$. A combination of the film formation and rupture maps can be used to estimate the existence of liquid film on the membrane. For example, for flow boiling with constant mass flux, a film may form with increasing quality due to flow acceleration resulting from phase change along the flow. As the quality increases the liquid phase may be insufficient to form a film as predicted from Eqs. (5) and (6).

2.2.3. Stratified to intermittent flow regime transition

The criterion for two-phase flow regime transition from stratified to intermittent flows is used here to identify the transition from vapor only to two phase contact with the membrane. The criterion developed by Taitel and Dukler [28] is used in this study, but is modified for rectangular cross section channels, where the lift, caused by vapor acceleration over a liquid–vapor wave interface, disturbs the interface leading to the rapid growth of interfacial waves which may then block the vapor flow path. There are two conditions to be satisfied. First, the lift, due to the decreased pressure of the accelerating vapor over the wave interface overcomes the damping gravitational force. Second, the liquid phase mass fraction is sufficient to maintain slug or annular flow.

By considering a finite solitary wave, Taitel and Dukler [28] expressed the condition for rapid wave growth in terms of the required average vapor velocity as:

$$u_x > C_w \left( \frac{P_l - P_v}{\rho_v} \right) \frac{g \cos \beta}{\rho_v} \frac{A_c}{dA_c/dH_l} \frac{1}{\sqrt{2}}$$  \hspace{1cm} (7)

where $C_w = A_{c0}/A_{c}$, is the ratio of is the cross-sectional area of vapor flow, $A_{c0}$, and the cross sectional area of the finite wave peak $A_c$, and ranges between 0 and 1. The rapid wave growth condition can be rewritten as a function of modified Froude number, $Fr'$, for a general channel geometry as:

$$Fr' > C_w \left( \frac{A_{c0}}{A_c} \right)^3 \frac{A_{c0}/H_l}{dA_c/dH_l}$$  \hspace{1cm} (8)

where $Fr'$ is defined as:

$$Fr' = \sqrt{\frac{\rho_v}{\rho_l - \rho_v} \frac{g l}{\sqrt{H_l} \cos \beta}}$$  \hspace{1cm} (9)

For rectangular channel, Eq. (8) is rewritten as:

$$Fr' > C_w(1 - H_l/H_l)^{3/2}$$  \hspace{1cm} (10)

The area ratio $C_w$ is estimated by Taitel and Dukler [28] as the ratio of the liquid to channel depth:

$$C_w \approx 1 - \frac{H_l}{H}$$  \hspace{1cm} (11)

Substituting this estimated value of $C_w$, the criterion for rapid wave growth in a rectangular channel becomes:

$$Fr' > (1 - H_l/H_l)^{3/2}$$  \hspace{1cm} (12)

Based on the equilibrium stratified flow condition provided by Taitel and Dukler [28] the relative liquid height is a unique function of the Lockhart–Martinelli parameter $X^2$, denoted as:

$$X^2 = \frac{(dP/dz)_l}{(dP/dz)_v}$$  \hspace{1cm} (13)

In addition, for non-horizontal flows the gravitational influences are represented by the nondimensional parameter $Y$:

$$Y = \frac{(P_l - P_v)g \sin \beta}{(dP/dz)_v}$$  \hspace{1cm} (14)

The condition for an equilibrium stratified flow is developed by considering the momentum equation of each phase and then equating the pressure drop terms of each. Using this condition, the criterion for rapid wave growth in circular tube is written as a function of $Fr'$, $X^2$ and $Y$. The resultant condition for equilibrium stratified flow as modified for any channel cross sectional geometry is:

$$X^2 \left( \frac{S_c}{S} \right)^{n+1} \left( \frac{A_c}{A_{c0}} \right) + Y - \left( \frac{S_c}{S} \right)^m \left( \frac{A_c}{A_{c0}} \right) \left( \frac{S_c}{S} \right)^{m+2} \left( \frac{A_c}{A_{c0}} \right)$$

$$+ \left( \frac{S_c}{S} \right)^m \left( \frac{A_c}{A_{c0}} \right) = 0$$  \hspace{1cm} (15)

The exponents $m$ and $n$ are the exponents of the Reynolds number in the Blasius equation for vapor and liquid phases, respectively, which are equal to 1.0 for laminar flow and 0.2 for turbulent flow. Note that for laminar–laminar or turbulent–turbulent flow boiling ($n = m$), $X^2$ can be rewritten as a function of quality, $x$, as:

$$X^2 = \left( \frac{H_l}{H} \right)^n \left( 1 - x \right) \left( \frac{v_l}{v_v} \right)^{2-n}$$  \hspace{1cm} (16)
The cross-sectional area, $A_c$, is dependent on channel geometry as shown in Fig. 3. For a rectangular channel, $S$ and $A_c$ in Eq. (15) are evaluated as a function of $H_l/H$ and channel aspect ratio.

\[
\frac{A}{A_c} = (H_l/H)^{-1}
\]

(17)

\[
\frac{A}{A_c} = (1 - H_l/H)^{-1}
\]

(18)

\[
\frac{S_l}{S} = \frac{2H_l/H + \alpha_c}{2(1 + \alpha_c)}
\]

(19)

\[
\frac{S_l + \alpha_c}{S} = 1 - \frac{H_l/H}{1 + \alpha_c}
\]

(20)

\[
\frac{S_l}{S} = \frac{\alpha_c}{2(1 + \alpha_c)}
\]

(21)

The condition for rapid wave growth in a rectangular channel is evaluated as a function of $Fr^2$, $\alpha_c$, $X^2$, and $Y$ by combining Eqs. (10), (15), (17)-(21).

As previously mentioned, in addition to rapid wave growth, the liquid level must be sufficient to form a liquid slug. Taitel and Dukler [28] suggested that the transition between intermittent and annular flow takes place for:

\[
H_l/H > 0.5
\]

(22)

while in the later work by Barnea et al. [33] recommend that the criterion for transition is:

\[
H_l/H > 0.35
\]

(23)

Using this latter criterion along with the condition for equilibrium stratified flow, Eqs. (15), (17)-(21), forms a relationship among $\alpha_c$, $X^2$, and $Y$ for regime transition.

It should be noted that Barnea et al. [29] suggests that the rapid wave growth and sufficient liquid level conditions, developed by Taitel and Dukler [28], is not sufficient to be applied to small channels. Rather, surface tension forces that attract liquid to the channel walls may need to be included. However, this additional criterion is not applied here since the membrane used for vapor extraction application is typically hydrophobic, with minimal surface tension forces, so this effect is expected to be minimal.

Based on the above approach, Fig. 4 shows the membrane contact phase for horizontal two-phase laminar–laminar flow in a rectangular channel without a liquid film formation using liquid level criterion from Barnea et al. [37]. The region on the right represents intermittent flow which corresponds to mixed phases in contact with the membrane. The sudden change in slope near $X^2 = 1$ is due to the change from rapid wave growth to the sufficient liquid level condition. This shows that for low values of $X^2$, i.e. high quality, the criterion is independent of $Fr^2$, because the liquid phase volume is not sufficient to form on the membrane. It also shows that the vapor-only region extends to larger $X^2$ values with increasing channel aspect ratio. This is due to the fact that as the channel becomes wider and there is larger contact area for the vapor phase. Therefore, the required quality to match the pressure drop of both phases in the channel is less, i.e. a higher value of $X^2$.

A dimensional membrane contact phase map is developed by combining conditions for liquid film formation with transition from stratified to intermittent flow. Two examples of this map are shown in Fig. 5. The lines denoted as A, B, C and D are based on film formation, Eq. (4), film rupture criteria, Eq. (5), rapid wave growth, Eq. (12), and sufficient liquid level, Eq. (23), respectively. Two versions are presented: Fig. 5(a) based on mass flux, $G$, versus quality, $x$, and Fig. 5(b) based on superficial liquid and vapor velocities. It is implied by observing Fig. 5(a) that full extraction is hard to achieve since near zero quality there is a “mixed phases” region which potentially reduces the effective bubble extraction area. Since extracting vapor decreases the available quality, the membrane contact phase changes from “vapor only” to “mixed phases” at around $x = 0.015$. This supports the observation by several studies [19,20] that vapor extraction from two-phase flow is less effective compared with single-phase membrane transport in some flow conditions. This membrane contact phase map can be used as a basis to develop models for vapor extraction from two-phase flow since the vapor extraction should be related to the relative area of the vapor phase in contact with the membrane.

2.3. Membrane transport mechanism

As previously discussed and summarized in Table 1 the conditions to initiate bubble extraction and evaporation are linked to the following pressure conditions:

\[
P_{\text{bub}} > P_{\text{extr}}
\]

(24)

and

\[
P_{\text{extr}} > \Delta P_{\text{break}}
\]

(25)

respectively, while the condition for liquid breakthrough is:

\[
P_{\text{chan}} - P_{\text{extr}} > \Delta P_{\text{break}}
\]

(26)

Since in microchannel flows bubble size will generally be on the order of the channel dimensions the difference in bubble and local channel pressure is assumed negligible. Therefore, the condition for bubble extraction becomes:

\[
P_{\text{chan}} > P_{\text{extr}}
\]

(27)

The breakthrough pressure may vary with temperature as it relies on the surface tension force. To account for this variation, assuming a near linear dependence on surface tension, the
breakthrough pressure may be written in term of the surface tension ratio and experimental breakthrough pressure at a specific temperature, $T_{\text{mem}}$, the breakthrough condition becomes:

\[ P_{\text{extr}} > P_{\text{chan}} - \frac{\sigma_{i\text{extr}}}{\sigma_{i\text{break}}} \Delta P_{\text{break}} \]

Using the conditions for evaporation, bubble extraction, and breakthrough, Eqs. (25), (27) and (28) the membrane transport mechanism can be determined based on channel pressure, membrane temperature and extraction pressure. For example, a membrane transport mechanism map for water at 100 kPa-a and membrane temperature and extraction pressure. For example, although the region at 75°C with 50 kPa-a extraction pressure in Fig. 6 is shown as “Evaporation/Bubble extraction”, only evaporation would occur if a liquid film is formed on the membrane.

2.4. Extraction flow regimes

Vapor extraction affects flow boiling, consequently flow conditions change and thereby so does the vapor extraction conditions. In other words, vapor extraction and flow boiling conditions are coupled. In this section, vapor extraction affected by flow boiling is characterized as the extraction flow regime based on the extraction mechanism regimes (b)–(e) shown in Table 2.

Fig. 6. Membrane transport mechanism map for water at 100 kPa-a and membrane breakthrough pressure of 70 kPa, at 25°C, lines A, B and C represent transition criteria for evaporation, bubble extraction and liquid breakthrough, using Eqs. (25), (24) and (28), respectively.

Six extraction flow regimes are identified by criteria based on the amount of vapor leaving the outlet of the channel, the boiling condition, and flow stability. These are: (i) single-phase evaporation, (ii) two-phase evaporation – bubble collapse, (iii) full extraction – stable, (iv) full extraction – unstable, (v) partial extraction – stable and (vi) partial extraction – unstable. The phenomenological conditions of each regime are summarized in Table 3 where the characteristics of each regime are described as:

i. Single-phase evaporation is when there is no vapor generation within the channel, but evaporation occurs through the membrane.
ii. Two-phase evaporation – bubble collapse occurs during subcooled boiling, and the generated bubbles are condensed in the flow rather than being extracted, concurrently evaporation occurs at the membrane.
iii. Full extraction – stable is when all generated bubbles are extracted and the flow is stable.
iv. Full extraction – unstable is when all generated bubbles are extracted, but the vapor extraction process is not sufficient to stabilize the flow.
v. Partial extraction – stable has some generated bubbles extracted, leaving excess vapor exiting through the channel outlet while the flow is stable.
vi. Partial extraction – unstable is similar to regime (v) but the flow is unstable.

2.5. Extraction flow regime transition criteria

Similar to the method used to identify the extraction mechanism regime transition discussed previously, the transition criteria of flow dynamics are not developed directly. Instead, the physical conditions related to extraction flow regimes, which are outlet quality, boiling condition and flow stability, are analyzed below. The extraction flow regimes are then identified by matching the physical conditions with the phenomenological conditions for each extraction flow regime, as summarized in Table 3.

2.5.1. Outlet quality

The quality at the channel outlet can be evaluated by considering conservation relationships applied globally to the entire channel as shown in Fig. 7. Combining conservation of mass and energy and neglecting kinetic energy terms results in:

\[ q + m_{\text{in}}(i_{\text{in}} - i_{\text{out}}) = m_{\text{extr}}(i_{\text{extr}} - i_{\text{out}}) \]

The relationship is approximated by assuming $c_p$, $i$, and $v$ are constant for the entire channel (neglecting pressure variations) and the enthalpy of the extracted vapor is estimated as the
The conservation equation for the condition of inlet subcooled liquid becomes:

\[ m_{in}(i_{in} - h) = q - m_{in}c_p\Delta T_{sub,in} - m_{extr}(i_p - (i_{out} - h)) \]  

(30)

or

\[ \frac{(i_{out} - h)}{i_p} = \frac{\frac{q}{m_{in}} - c_p\Delta T_{sub,in} - m_{extr}}{m_{in}} \left[ 1 - \frac{(i_{out} - h)}{i_p} \right] \]  

(31)

The quality at the channel outlet is:

\[ x_{out} = \frac{(i_{out} - h)}{i_p} \]  

(32)

or, using Eq. (31) is:

\[ x_{out} = \frac{\frac{q}{m_{in}} - c_p\Delta T_{sub,in} - m_{extr}}{m_{in}} \left( 1 - \frac{m_{extr}}{m_{in}} \right) \]  

(33)

which can be rewritten in terms of an extraction number, \( N_{extr} \), the ratio of the extracted and inlet mass flow rates and \( x_{out} \), the quality at the channel outlet that would occur without vapor extraction as:

\[ x_{out} = x_{out}^* - \frac{N_{extr}}{1 - N_{extr}} \]  

(34)

where \( x_{out}^* \) is:

\[ x_{out}^* = \frac{\frac{q}{m_{in}} - c_p\Delta T_{sub,in}}{i_p} \]  

(35)

It should be noted that \( x_{out}^* \) can be rewritten in terms of the modified boiling number, \( B_0^* = \frac{q}{m_{in}c_p} \), and Jacob number, \( j_a_{in} = \frac{\rho_s}{\rho_l} \frac{u_{sat}}{u_{in}} \), as:

\[ x_{out}^* = B_0^* - \left( \frac{\rho_s}{\rho_l} \right) j_a_{in} \]  

(36)

The relationship given by Eq. (34) is shown in Fig. 8. As can be seen, the quality at the channel outlet decreases with increasing \( N_{extr} \), since both energy and mass are extracted through the membrane. Note that vapor extraction can decrease the quality at the outlet to be negative, i.e. the outlet is a subcooled liquid, due to excessive evaporation when the extraction mass flow rate is greater than generated vapor (\( N_{extr} > x_{out}^* \)).

### Table 3: Phenomenological conditions of extraction flow regimes.

<table>
<thead>
<tr>
<th>Regime</th>
<th>Phenomenological conditions</th>
<th>Boiling condition</th>
<th>Flow stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>i. Single-phase evaporation</td>
<td>( x_{out} &lt; 0 )</td>
<td>No boiling</td>
<td>Stable</td>
</tr>
<tr>
<td>ii. Two-phase evaporation (bubble collapse)</td>
<td>( x_{out} &lt; 0 )</td>
<td>Subcooled boiling</td>
<td>Stable</td>
</tr>
<tr>
<td>iii. Full extraction (stable)</td>
<td>( x_{out} = 0 )</td>
<td>Saturated boiling</td>
<td>Stable</td>
</tr>
<tr>
<td>iv. Full extraction (unstable)</td>
<td>( x_{out} &gt; 0 )</td>
<td>Saturated boiling</td>
<td>Unstable</td>
</tr>
<tr>
<td>v. Partial extraction (stable)</td>
<td>( x_{out} &gt; 0 )</td>
<td>Saturated boiling</td>
<td>Stable</td>
</tr>
<tr>
<td>vi. Partial extraction (unstable)</td>
<td>( x_{out} &gt; 0 )</td>
<td>Saturated boiling</td>
<td>Unstable</td>
</tr>
</tbody>
</table>

![Fig. 7. Schematic of a channel control volume illustrating mass and energy transport.](image)

![Fig. 8. Extraction number, \( N_{extr} \), versus \( x_{out}^* \) for a range of desired outlet quality, \( x_{out} \).](image)

2.5.2. Boiling condition

Incipience of boiling occurs when the wall superheat, \( \Delta T_{sat} = T_w - T_{sat} \), is sufficient such that the local saturation pressure exceeds the combined static and surface tension induced pressures. Many studies have found that this wall superheat requirement depends on the size of active nucleation sites. For example, the nucleation criterion developed by Hsu [38] shows a range of active nucleation size as a function of wall superheat as:

\[ r_{c,min}, r_{c,max} = \frac{\delta_v \sin \theta_{c,w}}{2(1 + \cos \theta_{c,w})} \left( \frac{\Delta T_{sat,ONB}}{\Delta T_{sat,ONB} + \Delta T_{sub,ONB}} \right) \]  

(37)

where the thermal boundary layer thickness, \( \delta_v \), is estimated as \( k_t/h \). Generally, for a system with sufficient active nucleation sites, the minimum required wall superheat for the incipience of boiling can be simplified to:

\[ \Delta T_{sat,ONB} = \sqrt{\frac{8C_b\sigma T_{sat}q_{sw}}{\rho_l h_{lw}h_{lw}}} \]  

(38)

where \( C_b \) was initially proposed by Hsu [38] for pool boiling to be:

\[ C_b = 1 + \cos \theta_{c,w} \]  

(39)

Several investigators have modified the variable \( C_b \), examples are given in Table 4.

To determine whether boiling occurs within the channel, the incipience of boiling criterion may be written in terms of quality. Consequently, the outlet quality can be used as a measure of the occurrence of boiling, given in Eq. (34). Using definitions of the wall superheat and subcooling, \( \Delta T_{sub} = T_{sat} - T_w \), then:

\[ \Delta T_{sub} = \frac{q_{sw}}{h} - \Delta T_{sat} \]  

(43)

For all liquid flow up to the point of the onset of nucleate boiling, the all-liquid heat transfer coefficient, \( h_{iw} \), is:
At the inception of boiling Eq. (43) is then rewritten as:

\[ \Delta T_{\text{sub,ONB}} = \frac{q'}{h_b} - \Delta T_{\text{sat,ONB}} \]  

(45)

Multiplying this by \( \frac{q'}{m} \) and introducing the quality in the liquid region as:

\[ x = \frac{i - i_1}{i_u} = \left( \frac{c_p,i\Delta T_{\text{sub}}}{h_c} \right) \]  

(46)

the local value of quality at the onset of nucleate boiling is:

\[ x_{\text{ONB}} = \frac{c_p,i}{i_u} \left( \Delta T_{\text{sat,ONB}} - \frac{q'}{h_b} \right) \]  

(47)

For boiling to occur within the channel, the quality at the channel outlet must be greater than the quality for the onset of nucleate boiling, i.e. \( x_{\text{out}} > x_{\text{ONB}} \). Using the relationship among \( x_{\text{ONB}}, x_{\text{out}} \), and \( N_{\text{extr}} \), presented in Eq. (34), the boiling condition can be determined in terms of \( x_{\text{out}} \) and \( N_{\text{extr}} \). The criteria of the boiling conditions are summarized in Table 5. An example of the resultant boiling map using the minimum wall superheat correlation by Hsu [38] is shown in Fig. 9, indicating the subcooled and saturated boiling regimes.

### 2.5.3. Flow stability

The stability criterion applied to this study is based on whether vapor experiences reverse back flow into the inlet chamber. The criterion based on the model proposed by Salakij et al. [12], which was developed for flow boiling in a diverging channel with vapor extraction, is used in this study. The stability parameter, \( St \), that represents the ratio of backward to forward forces acting on the upstream side of the liquid–vapor interface of the expanding bubble, is expressed as:

\[ St = \sqrt{\frac{\rho_x \left( \frac{q_x h_{\text{hottub}} - m_{\text{extr}} C_{\text{vap}} h_{\text{hottub}}}{\rho_{\text{lv}} \Delta h} \right)^2}{\frac{\Delta P_{\text{extr}}}{\rho_x} + \sigma \nabla w \cos \theta}} \]  

(52)

where subscripts 1 and 2 represent the location of upstream and downstream sides of an expanding bubble, respectively. The transition from stable to unstable flow occurs when \( St = 1 \), and is evaluated here at the extreme case where an expanding bubble fills the channel and just reverses into the inlet chamber such that

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Boiling incipience models.</th>
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<tbody>
<tr>
<td>Author</td>
<td>Model</td>
</tr>
<tr>
<td>Sato and Matsumura [39] (pool boiling)</td>
<td>( C_b = 1 )</td>
</tr>
<tr>
<td>Ghiaasiaan and Chedester [40] (microchannel)</td>
<td>( C_b = 22^{0.765} ) where ( \xi = \frac{h_{\text{hottub}} - h_{\text{vap}}}{\rho_{\text{lv}} \Delta h} ) and ( R = \left( \frac{T_{\text{sat}} - T_{\text{lv}}}{\Delta h} \right)^{1/2} )</td>
</tr>
<tr>
<td>Kandlikar [41] (microchannel)</td>
<td>( C_b = 1 )</td>
</tr>
</tbody>
</table>

Fig. 9. Example of a boiling map for flow in 500 µm × 500 µm channel with length of 50 mm and contact angle of 53° where inlet mass flux and subcooling are 400 kg/m²s and 10 °C respectively.

\( \Delta h_{\text{hottub}} = \Delta h_{\text{cham}}, \ A_{\text{hottub}} = A_{\text{cham}}, \ A_{\text{1}} = A_{\text{in}}, \ \text{and} \ A_{\text{2,2}} = A_{\text{out}} \). It should be noted that this model does not account for the instability due to insufficient nucleation sites. Since the half-diverging angle of the channel, \( \theta_d \), is usually relatively small, on the order of 1°–3°, the term \( \cos \theta_d \) may be approximated as 1, and the criterion for stable flow, \( St < 1 \), is expressed as:

\[ \frac{q}{\mu_{\text{in}}} \left( 1 + \frac{A_{\text{out}}}{A_{\text{in}}} \right) \left( \frac{\rho_x}{\rho_i} \right) \left( 1 + 4 \frac{\rho_i \sigma D_i}{G_c D_i} \right)^{1/2} \left( \frac{m_{\text{extr}}}{m_{\text{in}}} \right)^{1/2} \]  

(53)

By dividing this by \( \frac{C_g A_{\text{in}}}{\rho_i} \) and using the inlet hydraulic diameter, \( D_{\text{in}} = \frac{4 A_{\text{in}}}{\pi} \), Eq. (53) becomes:

\[ \left( \frac{q}{\mu_{\text{in}}} \right) \left( 1 + \frac{A_{\text{out}}}{A_{\text{in}}} \right) \left( \frac{\rho_x}{\rho_i} \right) \left( 1 + 4 \frac{\rho_i \sigma D_i}{G_c D_i} \right)^{1/2} \left( \frac{m_{\text{extr}}}{m_{\text{in}}} \right)^{1/2} \]  

(54)

which can be rearranging as:

\[ \frac{q}{m_{\text{in}}} \left( 1 + \frac{A_{\text{out}}}{A_{\text{in}}} \right) \left( \frac{\rho_x}{\rho_i} \right) \left( 1 + 4 \frac{\rho_i \sigma D_i}{G_c D_i} \right)^{1/2} \left( \frac{m_{\text{extr}}}{m_{\text{in}}} \right)^{1/2} \]  

(55)

In dimensionless form, this criterion for stable flow is given in terms of the extraction number, \( N_{\text{extr}} \), the modified boiling number, \( Bo_{\text{in}}' \), and Weber number, \( We_{\text{in}} \), as:

\[ Bo_{\text{in}}' \left( 1 + \frac{A_{\text{out}}}{A_{\text{in}}} \right) \left( \frac{\rho_x}{\rho_i} \right) \left( 1 + 4 \frac{\rho_i \sigma D_i}{G_c D_i} \right)^{1/2} + N_{\text{extr}} \left( \frac{i_f}{i_u} \right)^{1/2} \]  

(56)

where the Weber number is defined as \( We_{\text{in}} = \frac{c^2 D_i}{\rho_i \sigma} \).

The stability map for water at 100 °C is shown in Fig. 10 where the left hand side of the line is the stable region. Fig. 10(a) shows that the stability region for a uniform cross-section channel is the smallest when \( We_{\text{in}} \) approaches infinity which represents negligible surface tension effects that suppress the instability compared with the inertia forces. This case is chosen to show the effect of varying cross-sectional area ratio, as shown in Fig. 10(b). In general, the stability region can be extended by increasing the cross-sectional area ratio (\( A_{\text{out}}/A_{\text{in}} \)) and \( N_{\text{extr}} \), and decreasing \( We_{\text{in}} \). This supports the results from several investigators [8–10] that expanding the channel improves flow stability.
3. Discussion

Ideally the completed extraction mechanism and extraction flow regime maps should be developed in the form of an independent single map. However, it may not be practical to combine all individual criteria together in this case because of the large number of independent variables to be included. For example, to get the complete single dimensional extraction mechanism map for a specific fluid flow in a specific channel geometry requires four independent variables: either \( (P_{\text{extr}}, T_{\text{mem}}, G, x) \) or \( (P_{\text{extr}}, T_{\text{mem}}, j_l, j_v) \). As previously suggested, it would be more practical to use individual maps to identify physical conditions of the regime, and then combine the known predicted conditions to identify the regime.

The regime transition criteria developed in this study are fully predictive based on physical concepts. The important assumptions used in the development are listed below:

- Film rupture when film thickness is in the same order as surface roughness.
- Membrane contact phase is strongly dependent on hydrodynamics of the flow where stratified flow and intermittent flows are related to vapor only and mixed phase contact, respectively.
- Transition from stratified to intermittent flow is caused by rapid wave growth due to the decreasing in pressure for vapor flow, and the liquid phase must be sufficient to form a slugs\(^2\).
- Membrane hydrophobicity prevents the surface tension force from pulling liquid to the top wall to form slugs.
- Negligible pressure differences occur between pressure inside the bubble and the surrounding liquid.
- For extraction flow analysis, fluid properties are assumed to be constant through the membrane.
- Enthalpy of the extracted vapor is estimated as the enthalpy of superheated vapor inside the extraction chamber.

Most of the assumptions and models used in this study have been validated and shown to be reliable for specific conditions in other types of applications. It is expected that to apply these models with reasonable adaptation to in-situ vapor extraction application, further modification is required. The models for regime transition criteria in this work are developed such that they can be modified to capture other effects.

To validate the assumptions and models for extraction mechanism regime transition, flow visualization on both sides of the membrane is required such that the membrane contact phase and breakthrough can be observed. Because an extraction mechanism regime depends on the local condition, the test section should be small and short such that the quality of flow boiling does not vary significantly within the observed section. The membrane surface temperature has to be measurable or, even better, controllable. Also, it is important to be able to have a precise quantitative measurement of film thickness that forms on the membrane and channel walls as well as surface roughness and contact angle of both membrane and channel walls. For extraction flow regime transition, precise measurements of mass flow rate, fluid temperature, pressure and quality at the inlet, outlet and extraction chamber are required as well as flow visualization to identify boiling mechanism and flow instability.

4. Conclusions

A systematic development of transition criteria for extraction mechanism and extraction flow regimes has been presented. All of the transition criteria are developed on a basis which can be easily modified to account for additional effects. Although the validation of the extraction mechanism and extraction flow regime transition criteria was not performed here, most models and assumptions have been validated in part in other applications. The methodology to validate transition models has been proposed and recommended for follow-on study. Examples of membrane contact phase, possibly extraction mechanism and stability maps, and effect of extraction on outlet quality are shown to be tools to identify regimes.

Conflict of interest

None declared.

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