

Model Predictive Control for Robots in Ocean Waves

As presented by:
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M.S. Candidate
Robotics

About Me

- 1st year in Coastal/Ocean Engineering
- Summer 2014: Hinsdale Wave Research Lab
- Supported CEOAS and OOI glider groups
- 2nd year with Robotic Decision Making Lab



- Ocean waves will displace a robot
- Wave disturbances lend to increased sensor drift
- Sensor drift reduces robotic observation quality
- Impending wave forces can be estimated
- **Objective: keep a station-keeping robot stationary under the influence of a wave field**

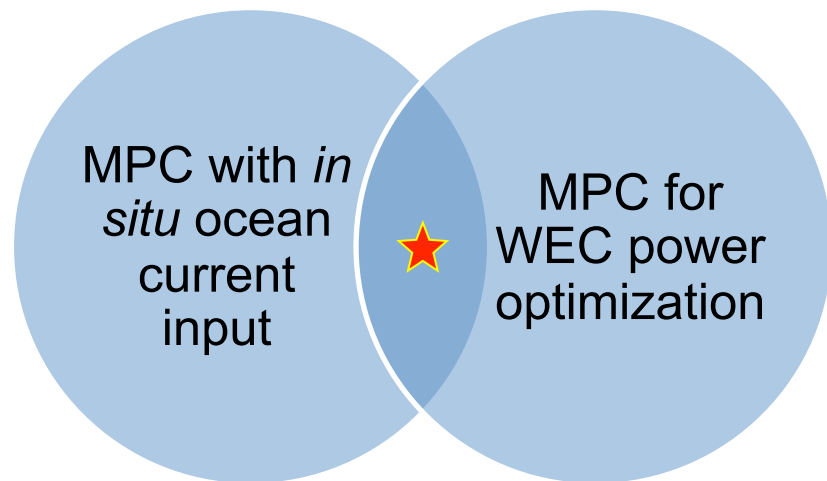


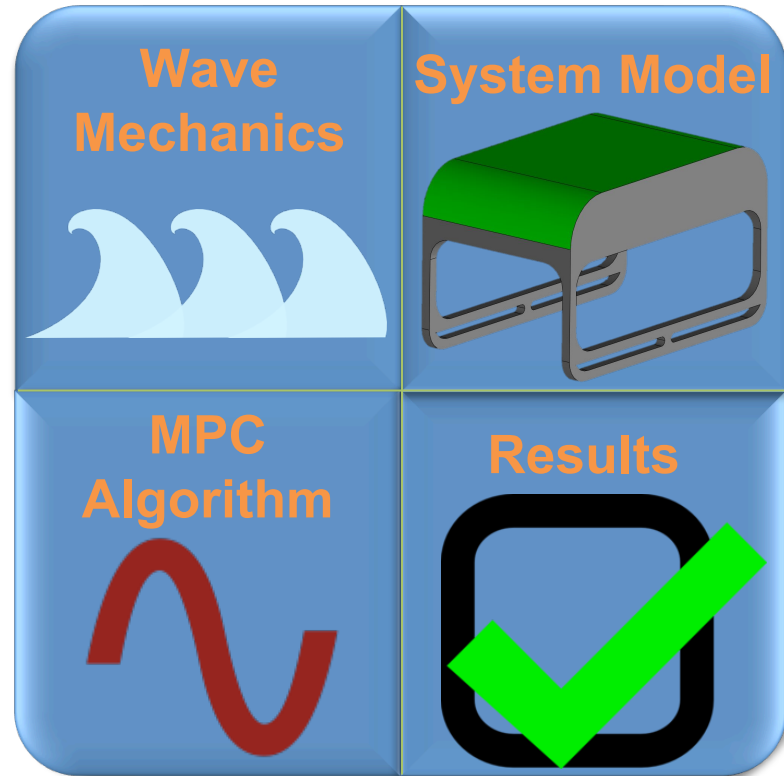
Source: *National Geographic*, 2012, R. D. Ballard

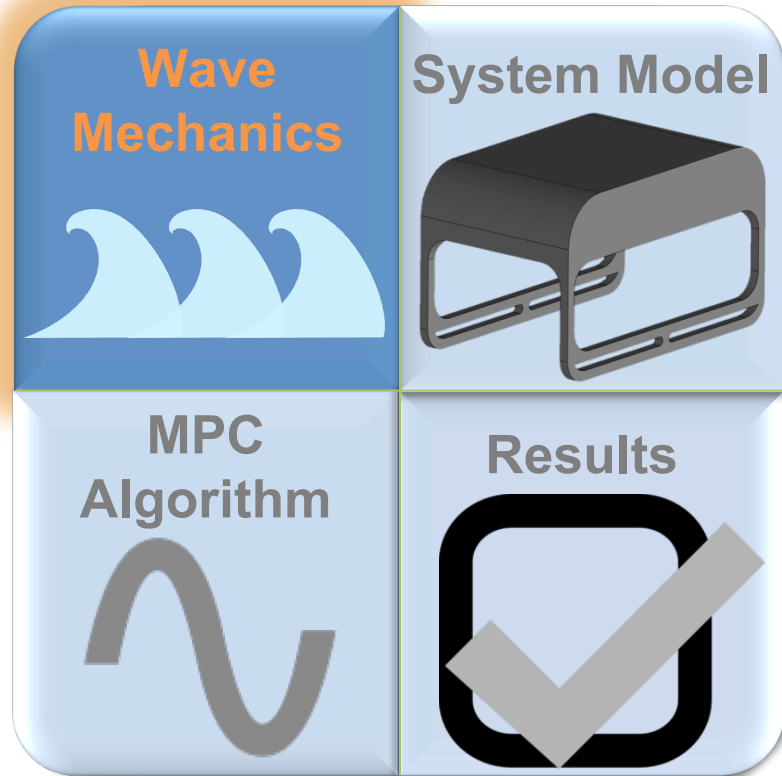
Motivation



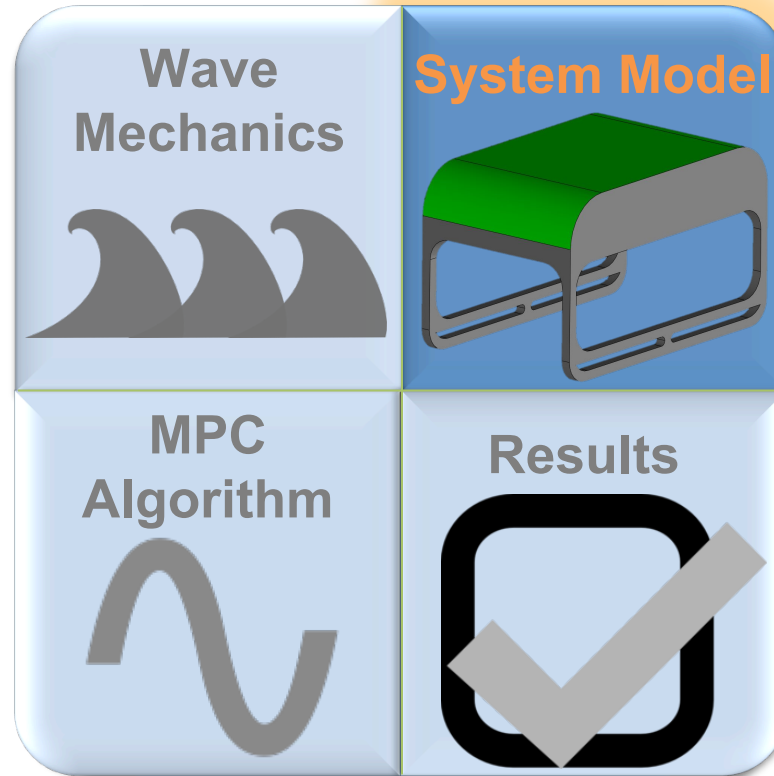
- Model Predictive Control (MPC)
 - Path planning with *in situ* ocean currents (Medagoda, 2012)
 - Wave Energy Converter (WEC) optimization (Brekken, 2011)
 - Station-keeping under water waves (Heidel, 1998)



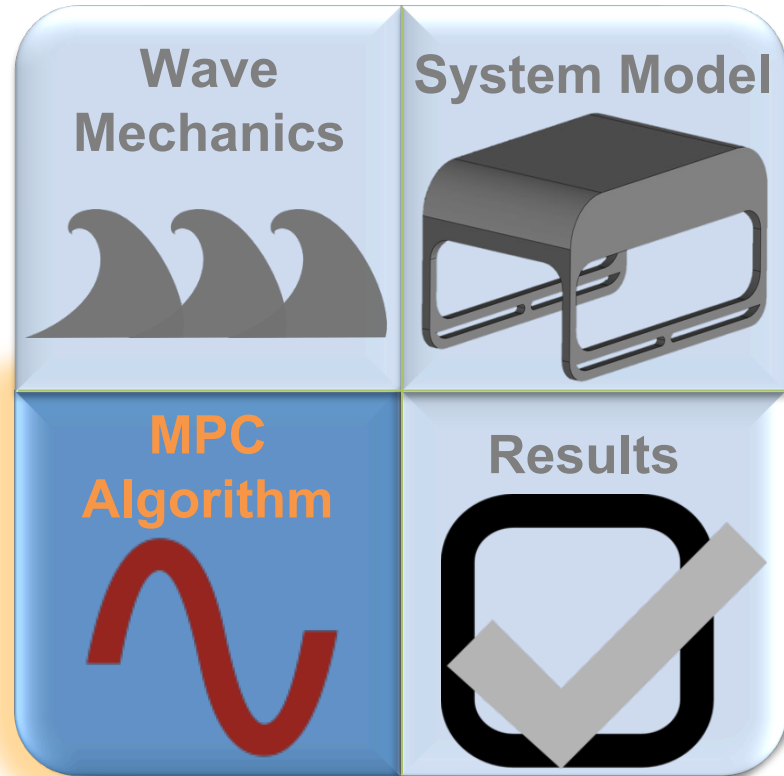




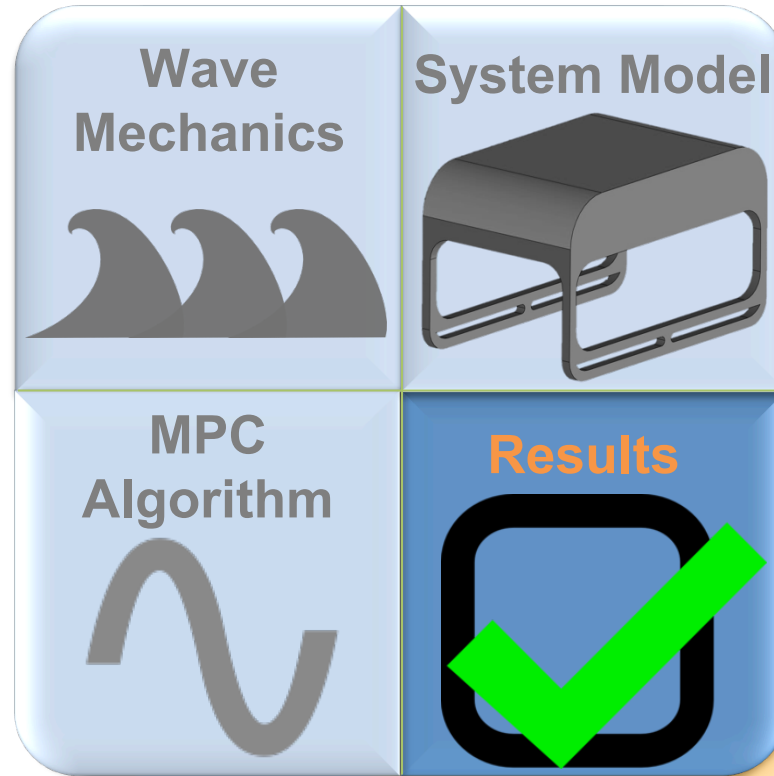
Outline

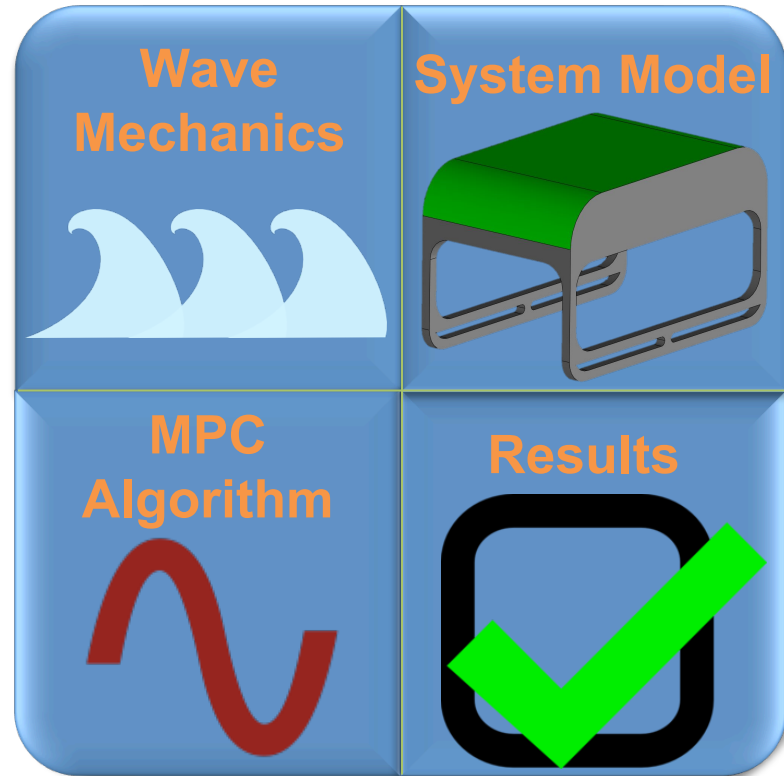


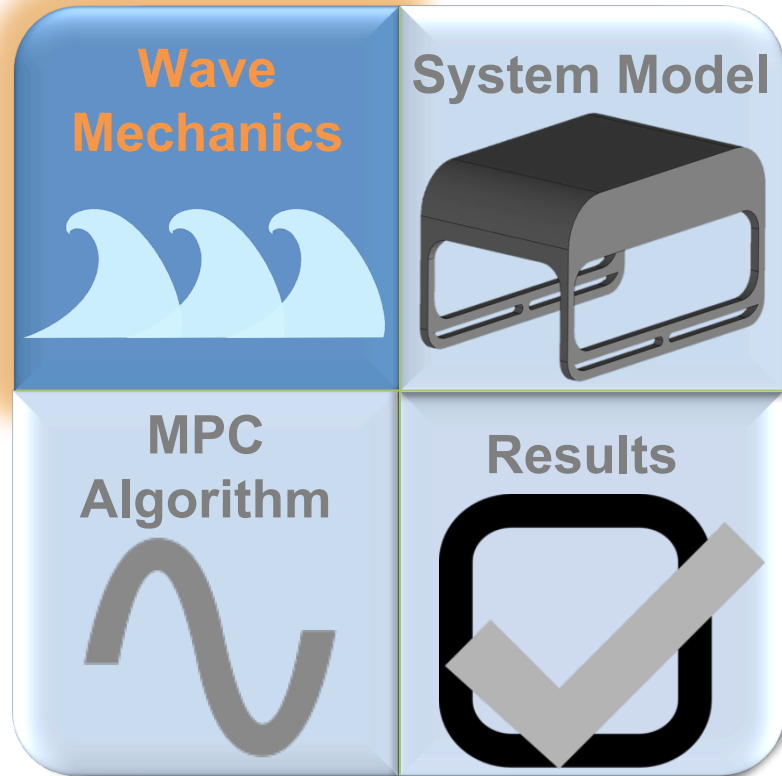
Outline



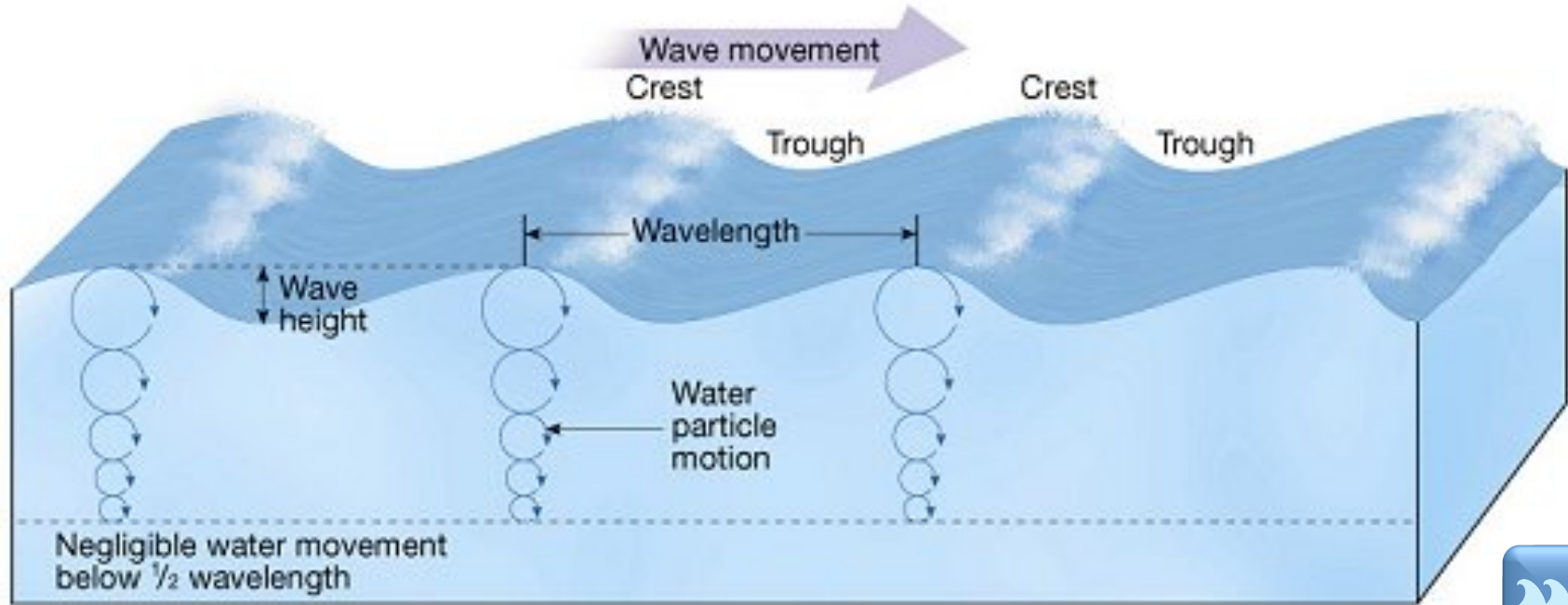
Outline







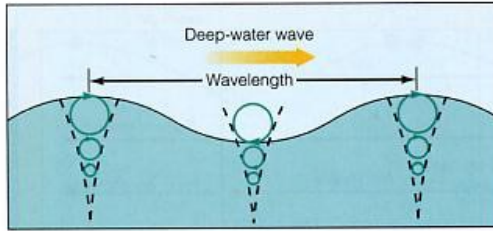
Parts of a Wave



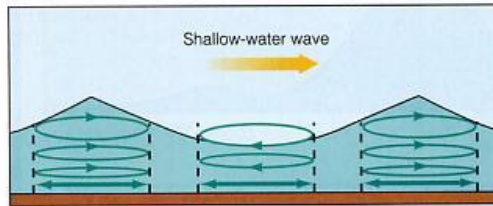
Source: *Waves, Tides and Shallow-Water Processes*, 1999, J. Wright, A. Colling, D. Park



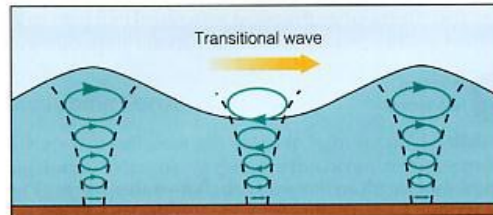
Water Motion under Waves



a Depth $\geq \frac{1}{2}$ wavelength



b Depth $\leq \frac{1}{20}$ wavelength



c $\frac{1}{20}$ wavelength \leq depth $\leq \frac{1}{2}$ wavelength

- Deep water wave

- circular paths, exponential decrease with depth
- $v/p = 1/2$ at $z = -L/9$, close to 0 at $z = -L/2$

- Shallow water wave

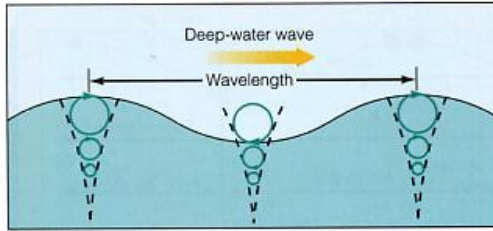
- More lateral motion than vertical
- More elliptical with depth, virtually lateral at depth

- Transitional water depths

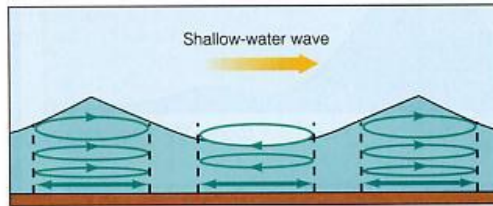
- Intermediate elliptical patterns
- Majority of waves in this work



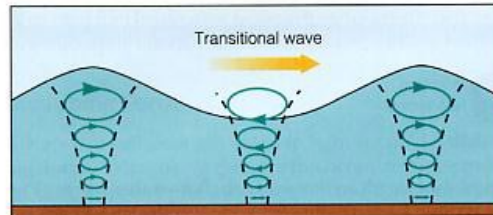
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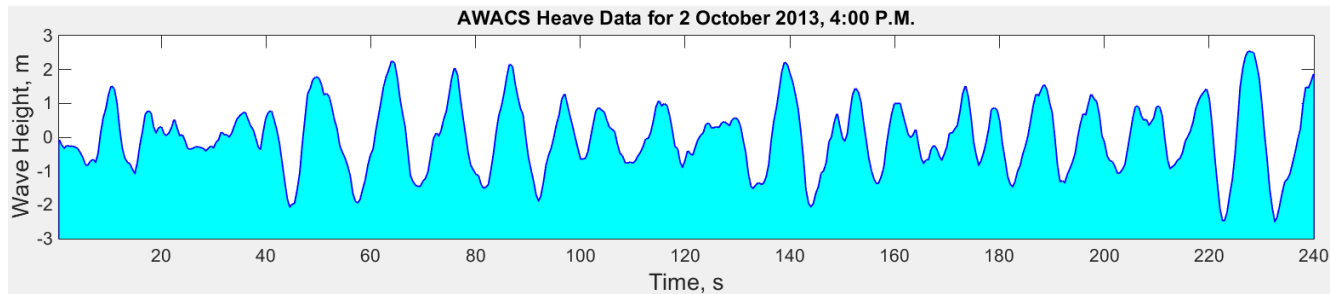
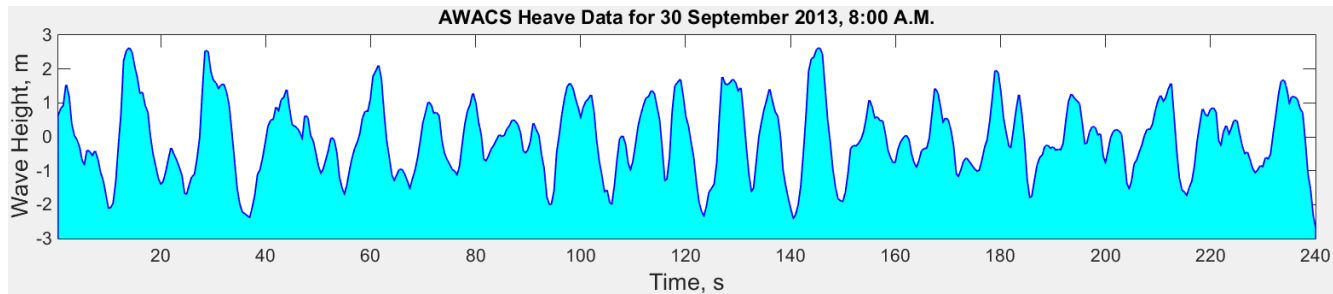
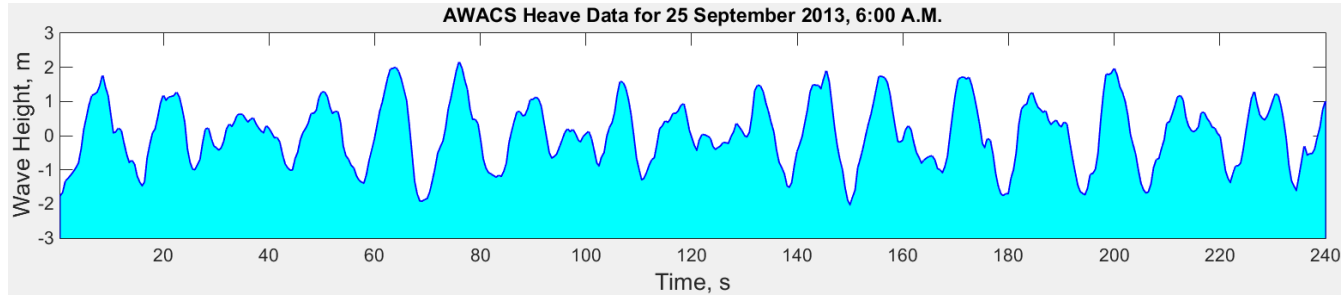


Simulation Setting (NETS)

- North Energy Test Site
- Operational depth: 50 m
- Validation of wave field
 - AWAC acoustic measurements
 - Deployed at NETS
 - August – October 2013
 - (600) 40 minute profiles (2 Hz)

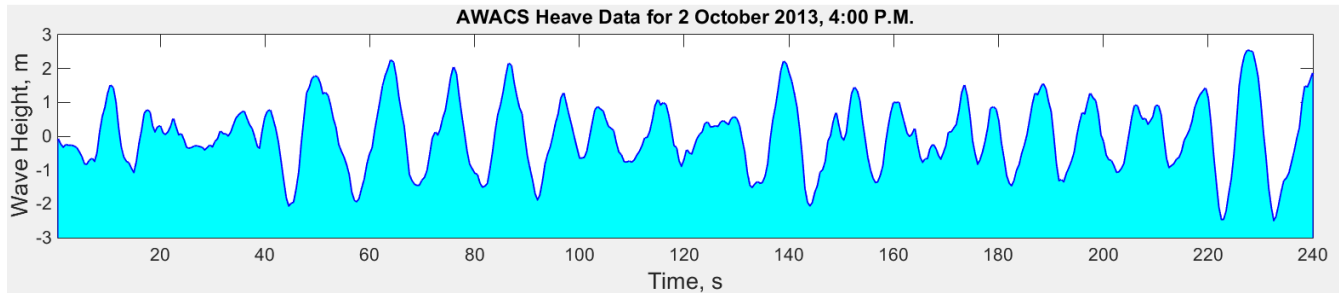


Simulation Setting (NETS)

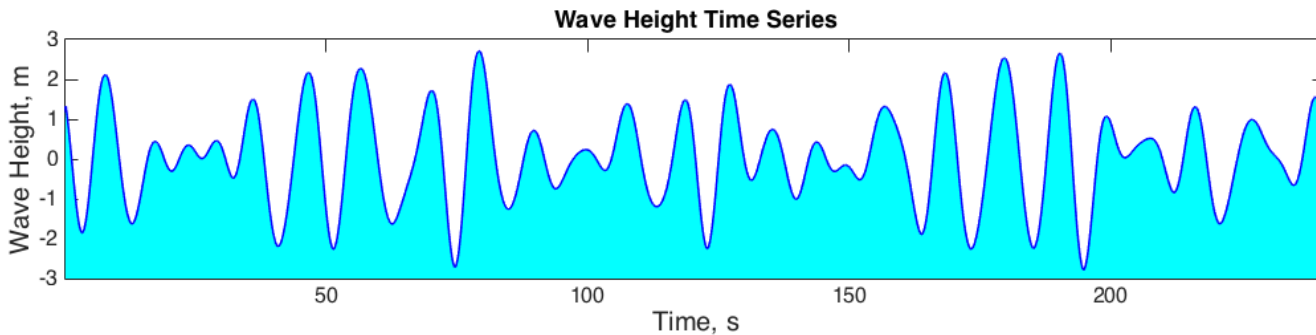


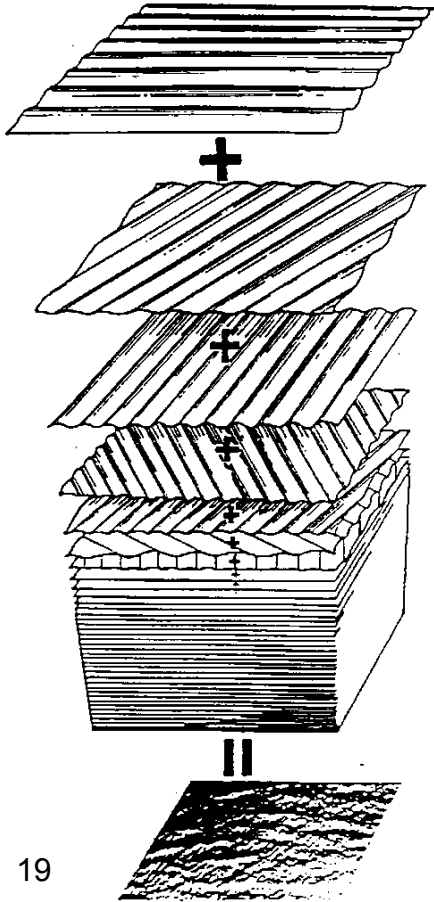
Simulation Setting (NETS)

Real Data



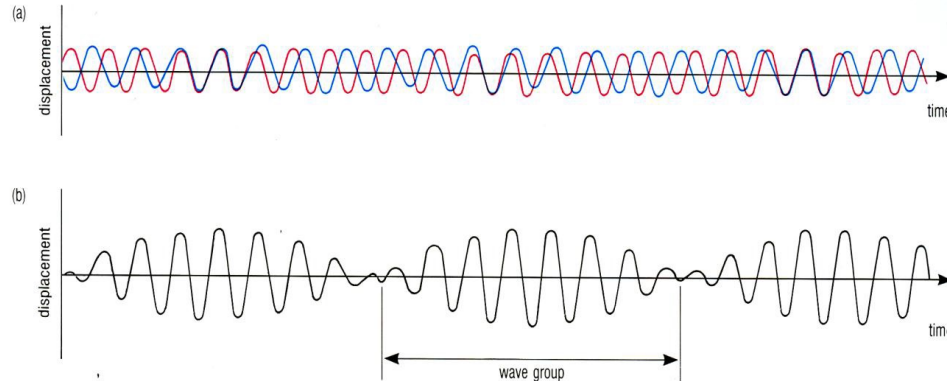
Simulated Data





- Superposition

- Sea surface can be represented by the sum of sinusoids with component periods (T), amplitudes (a), and phase offsets (ϕ)

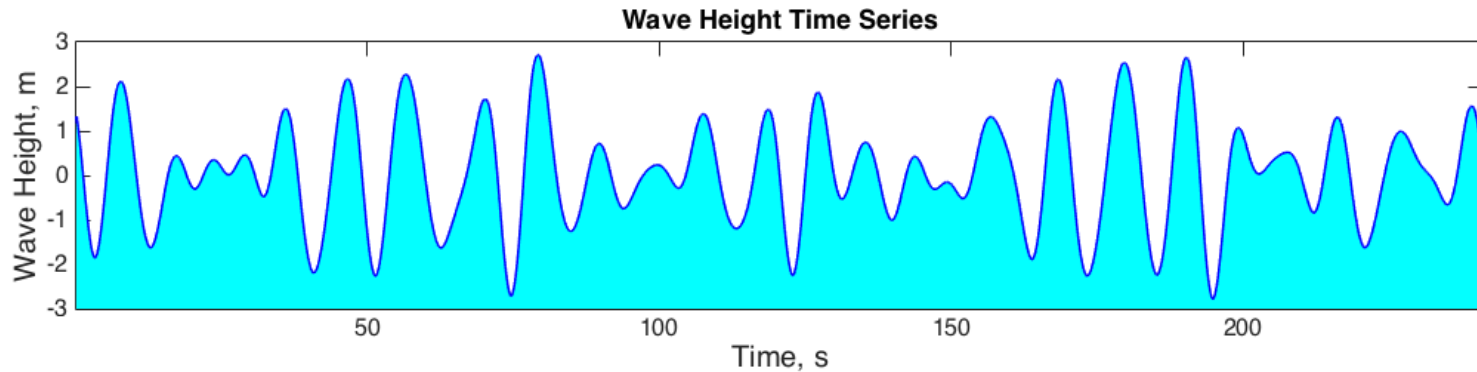


Source: *Waves, Tides and Shallow-Water Processes*, 1999, J. Wright, A. Colling, D. Park



Input Wave Field

Component Wave	1	2	3	4	5	6	7	8
Wave Period, T , s	10	8	12	11	6	7	9	25
Wave Height, H , m	1.8	0.9	1.6	1.3	0.4	0.5	1.1	0.7
Phase, ϕ , rad	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	$-\frac{5\pi}{8}$	$\frac{4\pi}{13}$	$-\frac{\pi}{15}$	$\frac{\pi}{3}$	$-\frac{\pi}{18}$	$-\frac{7\pi}{4}$



$$\eta(\mathbf{t}) = \sum \frac{H}{2} \cos(k\mathbf{x} - \omega\mathbf{t} + \phi)$$



Linear Wave Theory

- Assumes potential flow: $u = \nabla\phi$
- Dispersion Relation: $\omega^2 = gk \tanh(kd)$
- Wavelength: $L = gT^2 / 2\pi [\tanh((\omega^2 d/g)^{1/4})]^{2/3}$
- Wavenumber: $k = 2\pi/L$



- Lateral motion in transitional water

$$v_{p,x} = \frac{HgT}{2L} \frac{\cosh \frac{2\pi(z+d)}{L}}{\cosh \frac{2\pi d}{L}} \cos(kx - \omega t + \phi),$$

$$\dot{v}_{p,x} = \frac{g\pi H}{L} \frac{\cosh \frac{2\pi(z+d)}{L}}{\cosh \frac{2\pi d}{L}} \sin(kx - \omega t + \phi).$$



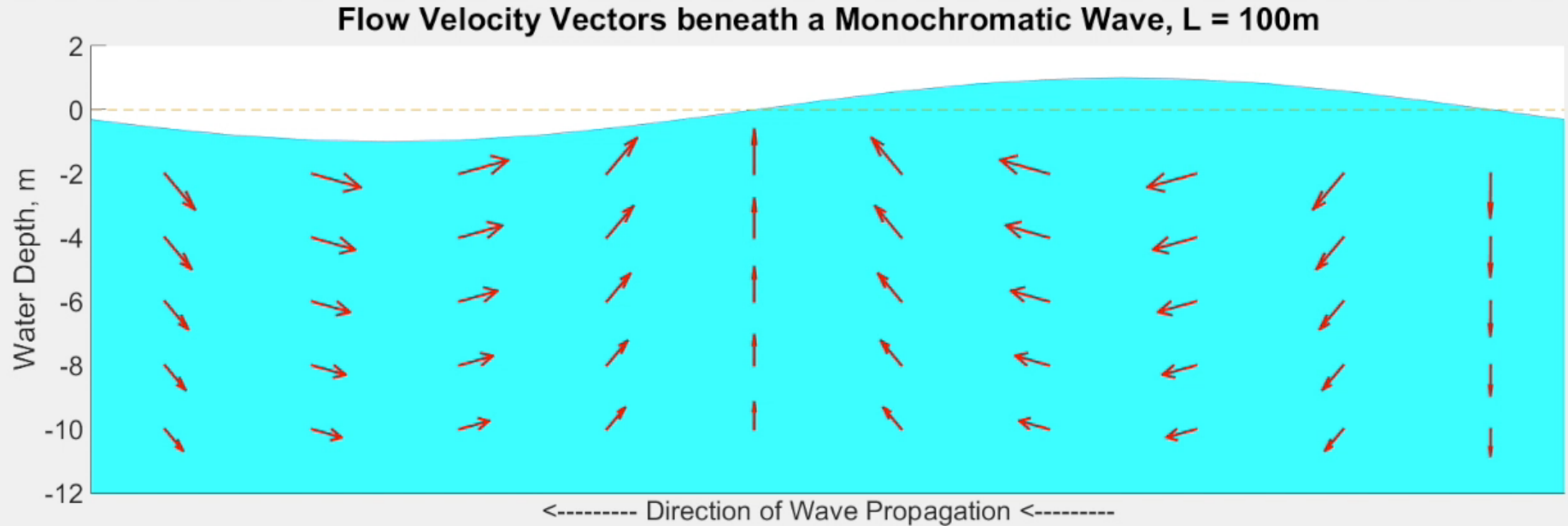
- Vertical motion in transitional water

$$v_{p,z} = \frac{HgT}{2L} \frac{\sinh \frac{2\pi(z+d)}{L}}{\cosh \frac{2\pi d}{L}} \sin(kx - \omega t + \phi),$$

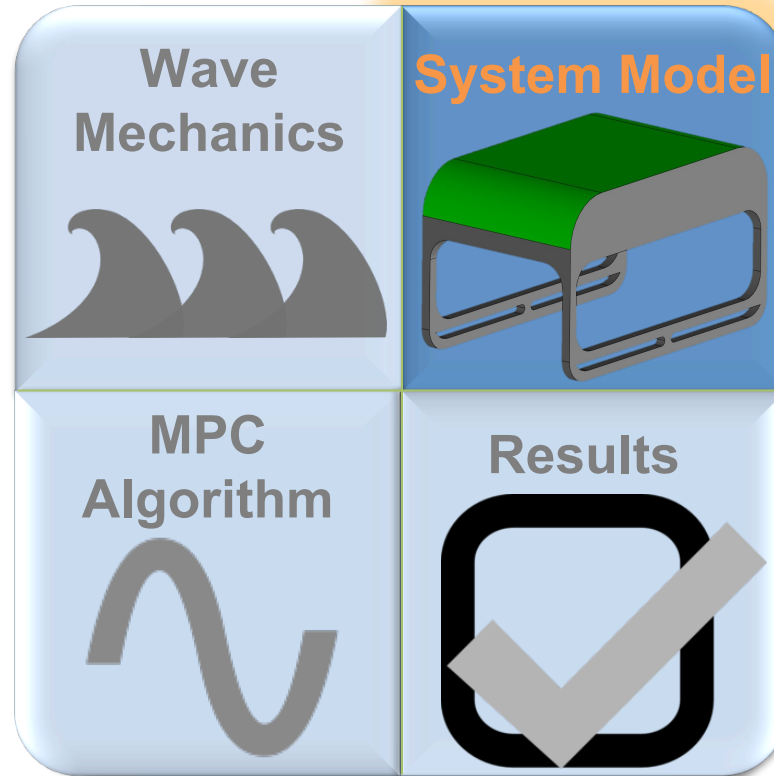
$$\dot{v}_{p,z} = -\frac{g\pi H}{L} \frac{\sinh \frac{2\pi(z+d)}{L}}{\cosh \frac{2\pi d}{L}} \cos(kx - \omega t + \phi).$$



Simulator (Fluid Motion)



Outline

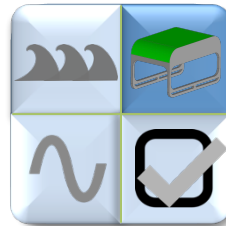


Remotely Operated Vehicle

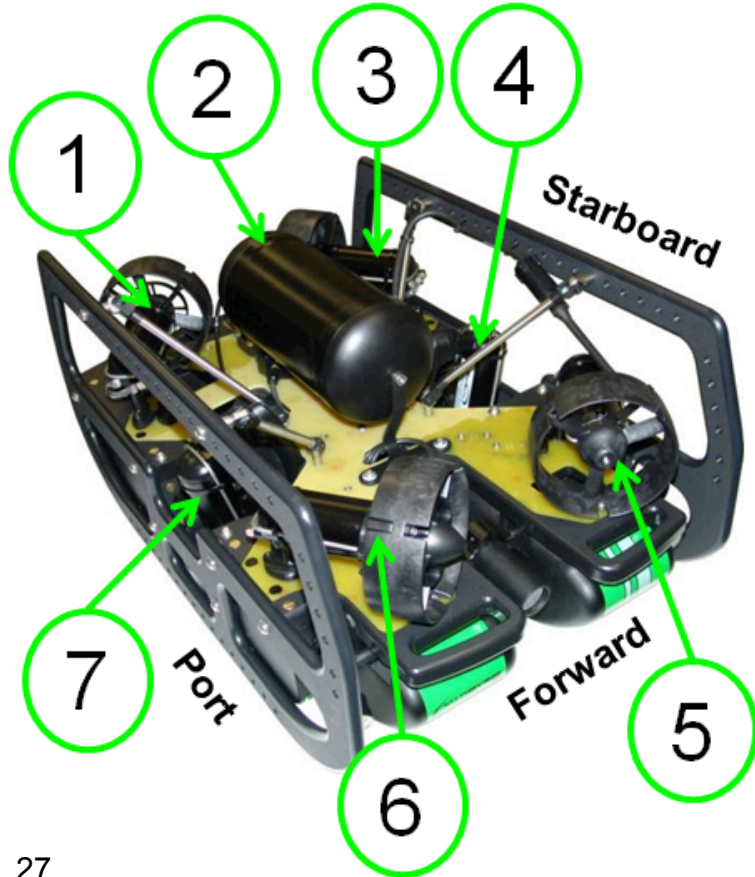
- SeaBotix vLBV300
 - Payload: 10kg
 - Depth Rating: 300m
 - Doppler Velocity Log (DVL)
 - Inertial Measurement Unit (IMU)
 - (6) 100mm brushless DC thrusters
 - Low light color camera with 180° vertical tilt



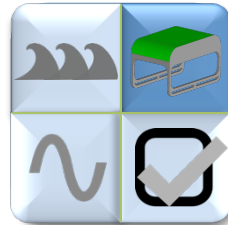
Source: Teledyne SeaBotix Inc.



Remotely Operated Vehicle

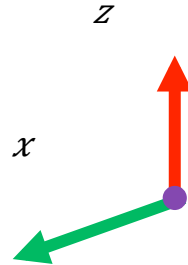
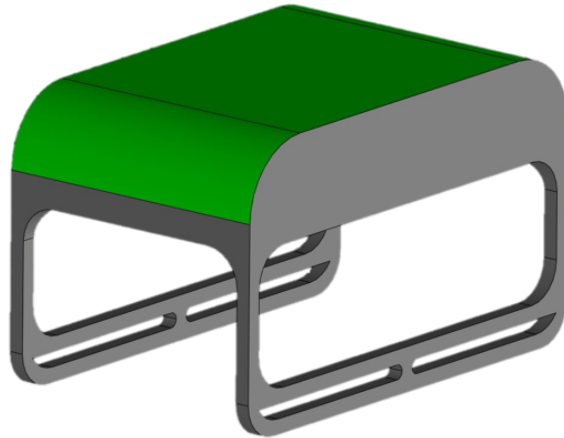


1. Port aft thruster
2. Electronics tube
3. Starboard aft thruster
4. Starboard vertical thruster
5. Starboard forward thruster
6. Port forward thruster
7. Port vertical thruster

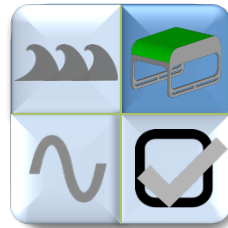


Remotely Operated Vehicle

- Dassault SolidWorks
- Ansys AQWA



Parameter	Symbol	Value
Density of Seawater	ρ_{sea}	1030 kg/m^3
Incident Area, x	$A_{i,x}$	0.156 m^2
Incident Area, z	$A_{i,z}$	0.273 m^2
Moment of Inertia, x	I_{xx}	0.62 kg m^2
Moment of Inertia, z	I_{zz}	1.60 kg m^2
Dry Mass	m_{dry}	22.2 kg
Added Mass, x	$m_{add,x}$	8.1 kg
Added Mass, z	$m_{add,z}$	36.7 kg
Drag Coefficient, x	$c_{d,x}$	0.84
Drag Coefficient, z	$c_{d,z}$	1.06
Max Thruster Force	T_{max}	29.7 N
Thruster Angle, Forward	θ_f	35°
Thruster Angle, Aft	θ_a	45°
Thruster Angle, Vertical	θ_v	20°

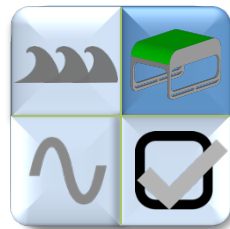


- System differential equation of motion:

$$\mathbf{M}\dot{\mathbf{v}}_a = \mathbf{F}_{thrust} + \mathbf{F}_d + \mathbf{F}_g + \mathbf{F}_c$$

- Simplify, sub inertia and drag relations:

$$m_{dry}\dot{\mathbf{v}}_a + m_{add}\dot{\mathbf{v}}_r = \mathbf{F}_{thrust} + \frac{1}{2}\rho_{sea}A_iC_d|\mathbf{v}_r|\mathbf{v}_r$$

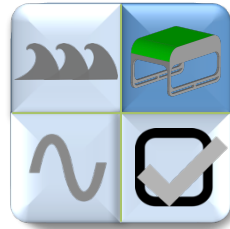


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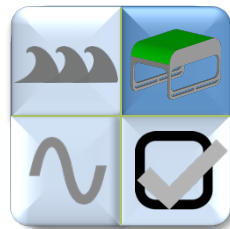
$$m_{dry}\dot{\mathbf{v}}_a + m_{add}\dot{\mathbf{v}}_r = \mathbf{F}_{thrust} + \frac{1}{2}\rho_{sea}A_iC_d|\mathbf{v}_r|\mathbf{v}_r$$



- Substitute v_{lp} such that $v_{la} = v_{lr} + v_{lp}$:

$$\begin{bmatrix} m_{dry} + m_{add,x} \\ m_{dry} + m_{add,z} \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} F_{thrust,x} \\ F_{thrust,z} \end{bmatrix} + \begin{bmatrix} m_{add,x} \\ m_{add,z} \end{bmatrix} \begin{bmatrix} \dot{v}_{p,x} \\ \dot{v}_{p,z} \end{bmatrix} + \frac{\rho_{sea}}{2} \begin{bmatrix} A_{i,x} C_{d,x} \\ A_{i,z} C_{d,z} \end{bmatrix} \begin{bmatrix} |\dot{x} - v_{p,x}| (\dot{x} - v_{p,x}) \\ |\dot{z} - v_{p,z}| (\dot{z} - v_{p,z}) \end{bmatrix}$$

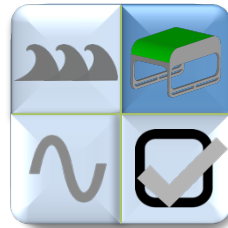
- where: $v_{la}, x=x$ and $v_{la}, z=z$



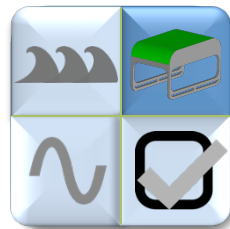
- In state space form:

$$\dot{\mathbf{r}} = \begin{bmatrix} \dot{x} & \ddot{x} & \dot{z} & \ddot{z} \end{bmatrix}^T = \mathbf{A}\mathbf{r} + \mathbf{B}\mathbf{u} + \mathbf{D}$$

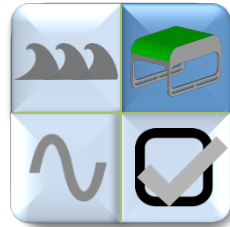
- where: $\mathbf{x} = \mathbf{v} \downarrow a, \mathbf{x}$ and $\mathbf{z} = \mathbf{v} \downarrow a, \mathbf{z}$



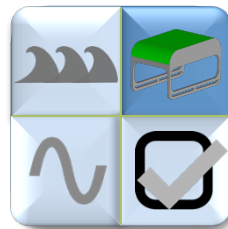
$$A\Upsilon = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ z \\ \dot{z} \end{bmatrix}$$



$$\mathbf{Bu} = \frac{T_{max}}{m_{dry}} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \cos\theta_f & \cos\theta_f & -\cos\theta_a & -\cos\theta_a & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\cos\theta_v & -\cos\theta_v \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix}$$

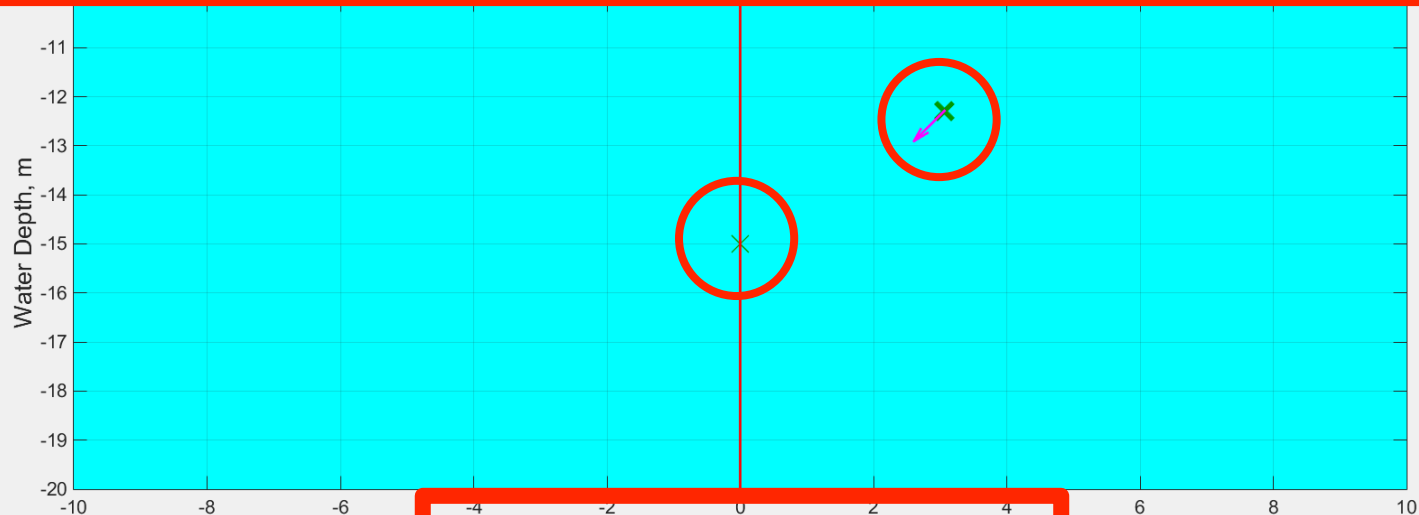
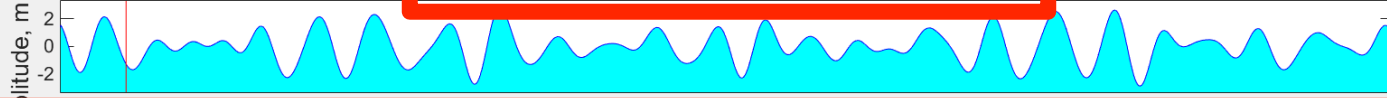


$$\mathbf{D} = \begin{bmatrix} 0 \\ \frac{\dot{\mathbf{v}}_{p,x}}{m_{dry}} + \frac{\rho_{sea} A_{i,x} C_{d,x}}{2(m_{dry} + m_{add,x})} |\dot{\mathbf{X}} - \mathbf{v}_{p,x}| (\dot{\mathbf{X}} - \mathbf{v}_{p,x}) \\ 0 \\ \frac{\dot{\mathbf{v}}_{p,z}}{m_{dry}} + \frac{\rho_{sea} A_{i,x} C_{d,z}}{2(m_{dry} + m_{add,z})} |\dot{\mathbf{Z}} - \mathbf{v}_{p,z}| (\dot{\mathbf{Z}} - \mathbf{v}_{p,z}) \end{bmatrix}$$



Simulator

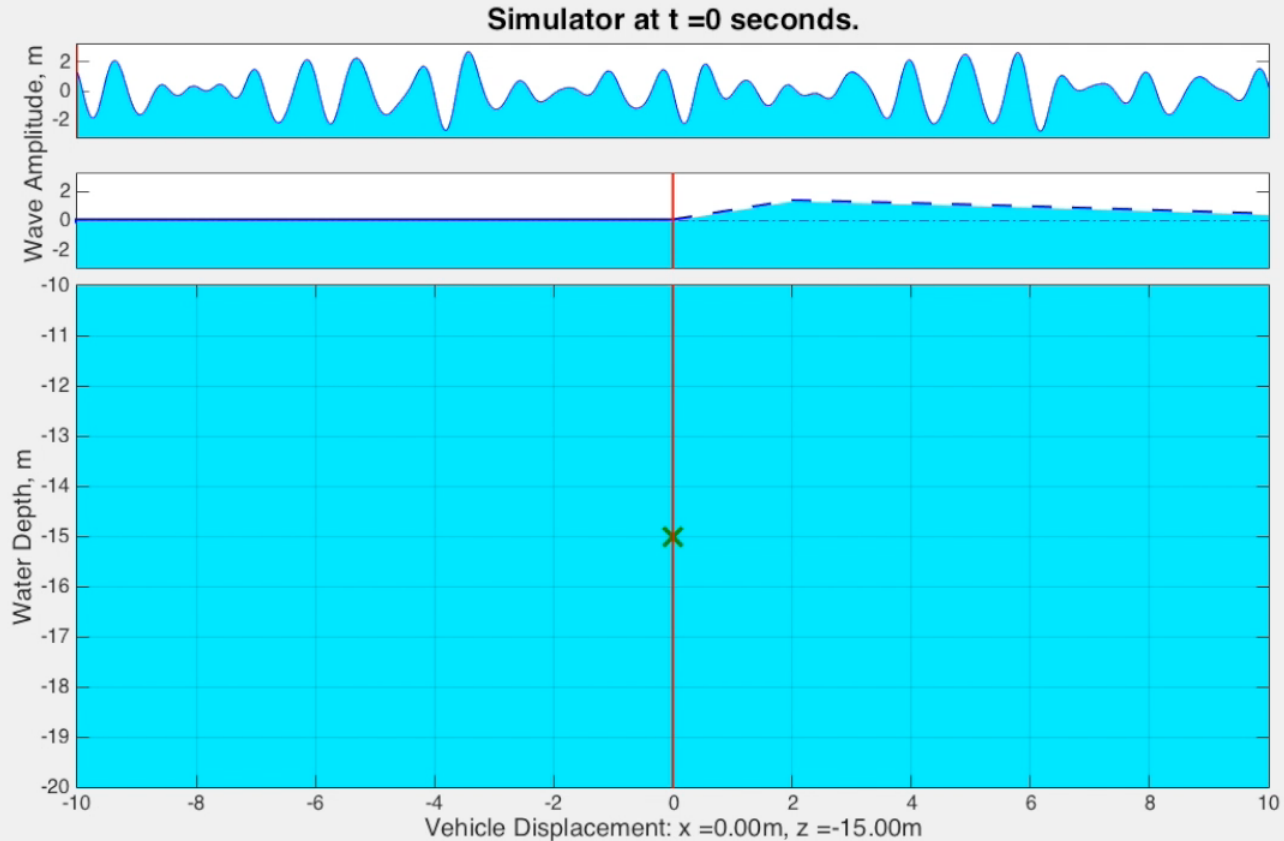
Simulator at $t = 12$ seconds



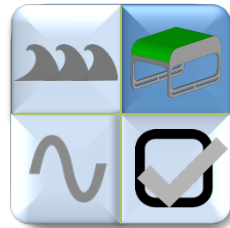
Vehicle Displacement: $x = 3.07$ m, $z = -12.29$ m



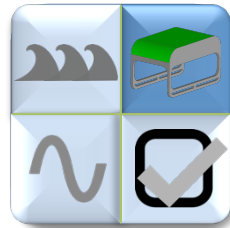
Simulator (Drifting Robot)



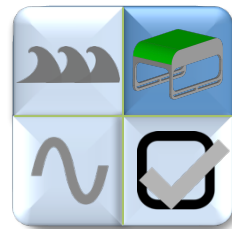
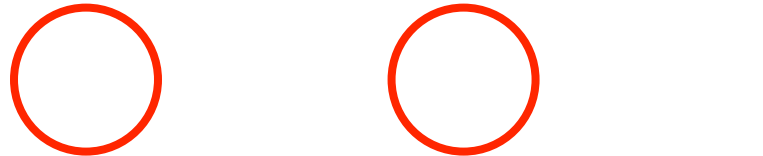
- Purely Reactive
- Use as base of comparison
- Position Derivative (PD) control



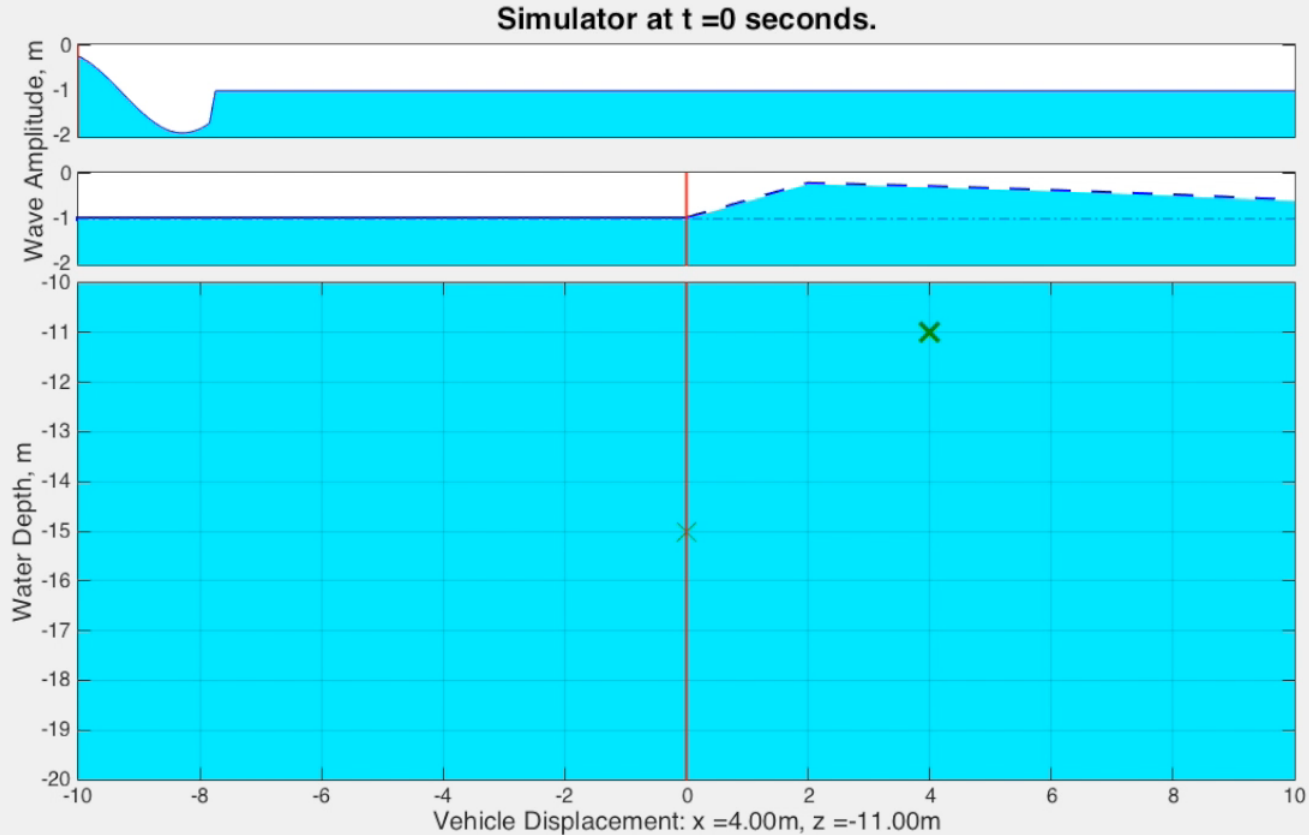
- Positional Error: $e_P = r_{target} - r_n$
- Derivative Error: $e_D = e_P, n - e_P, (n-1)$
- $[u_1 \ u_2 \ u_3 \ u_4]^T = K_P, x \ e_P, x + K_D, x \ e_D, x$
- $[u_5 \ u_6]^T = K_P, z \ e_P, z + K_D, z \ e_D, z$



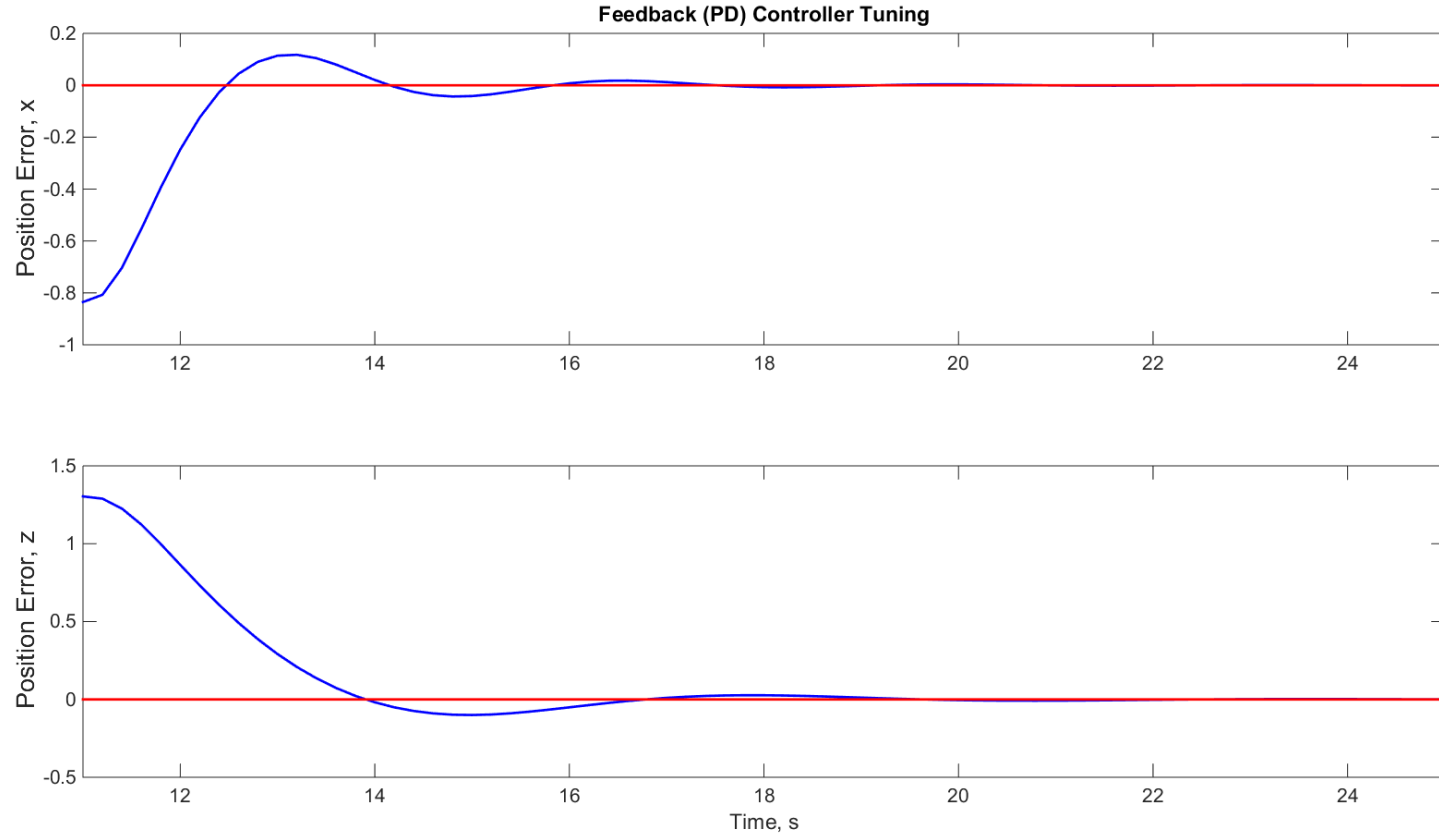
- Positional Error: $\epsilon_P = r_{target} - r_n$
- Derivative Error: $\epsilon_D = \epsilon_P, n - \epsilon_P, (n-1)$
- $[u_1 \ u_2 \ u_3 \ u_4]^T = K_P, x \epsilon_P, x + K_D, x \epsilon_D, x$
- $[u_5 \ u_6]^T = K_P, z \epsilon_P, z + K_D, z \epsilon_D, z$



PD Tuning



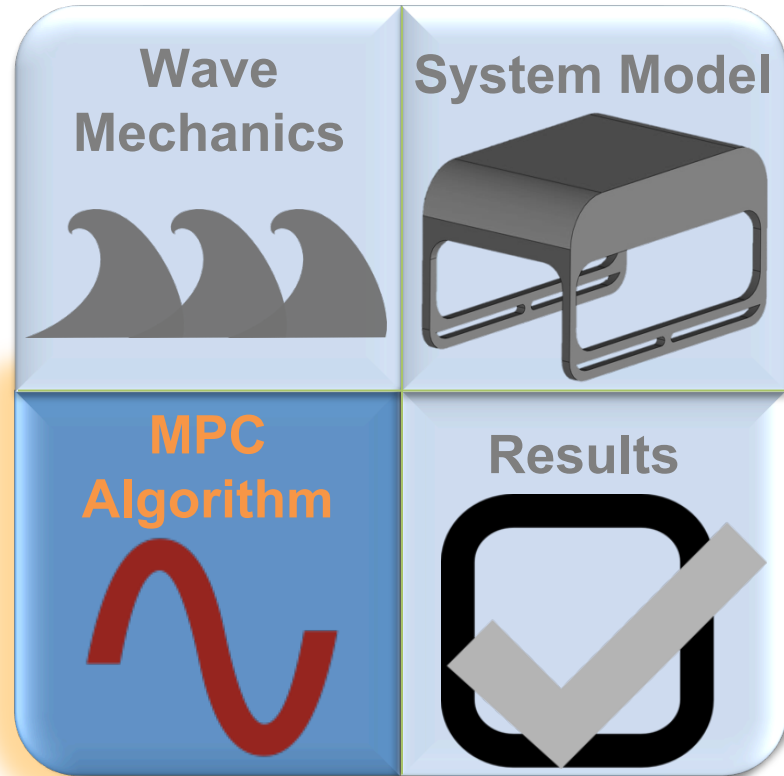
PD Tuning



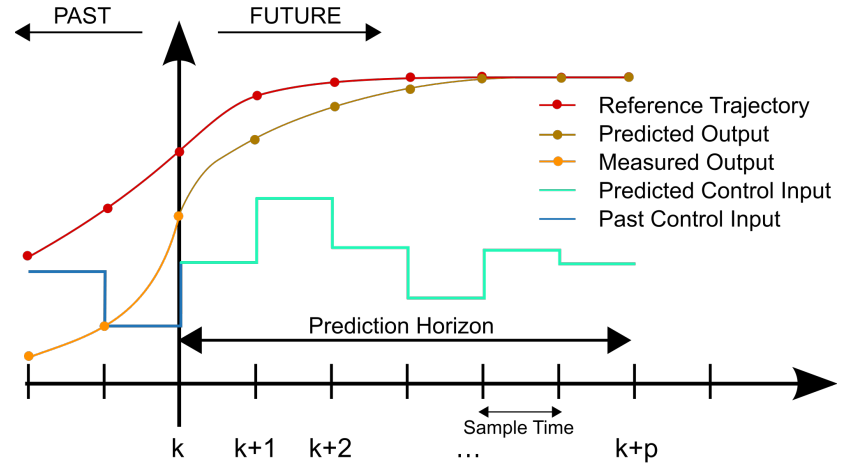
Simulator (PD Robot)



Outline



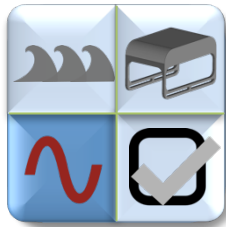
- State estimator
- Cost function
- Receding horizon
- **Objective:** Find the control actions that minimize the distance between the desired and predicted states.



Cost Function

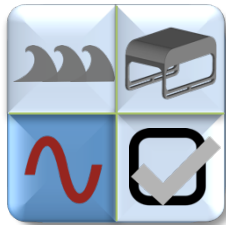
$$J = \sum_{k=1}^N [\mathbf{r}_{\text{target}} - \mathbf{r}_k(\mathbf{u}_k)]^2 + \beta \|\mathbf{u}_k\|^2$$

$$\mathbf{u}_{1:N}^* = \arg\min_{\mathbf{u}_{1:N}} J(\mathbf{u}_{1:N})$$



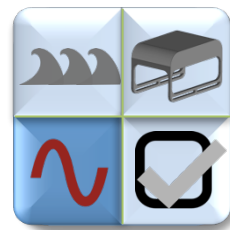
- Perturb new control input by Jacobian
 - Minimized as optimal control action is approached

$$\partial J / \partial \mathbf{u} = J_{\downarrow n} - J_{\downarrow n-1} / \mathbf{u}_{\downarrow n} - \mathbf{u}_{\downarrow n-1}$$



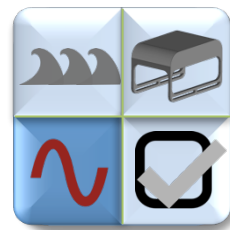
Algorithm 1 MPC for ocean wave station-keeping where t is a time vector, λ contains all relevant wave spectra data, and Υ represents states in time and space.

```
1: procedure  $MPC(t, \lambda, \text{robot}, \Upsilon_{target})$   
2:    $n \leftarrow 1$   
3:    $\eta \leftarrow \text{LOADSEASTATE}(t, \lambda, \Upsilon_{initial})$   
4:   while  $n < \text{simulatorOff}$  do  
5:      $\text{input} \leftarrow \text{GETFORECAST}(t, \text{robot}, \lambda, \Upsilon_{target}, n)$   
6:      $\text{robot} \leftarrow \text{MOVEROBOT}(t, \text{robot}, \lambda, \text{input}, n)$   
7:      $n \leftarrow n + 1$ 
```



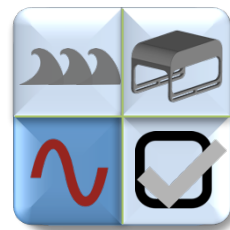
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5:      $\text{input} \leftarrow \text{GETFORECAST}(t, \text{robot}, \lambda, \Upsilon_{target}, n)$ 
6:      $\text{robot} \leftarrow \text{MOVEROBOT}(t, \text{robot}, \lambda, \text{input}, n)$ 
7:      $n \leftarrow n + 1$ 
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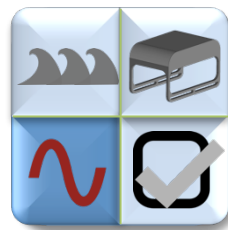
Algorithm 1 MPC for ocean wave station-keeping where t is a time vector, λ contains all relevant wave spectra data, and Υ represents states in time and space.

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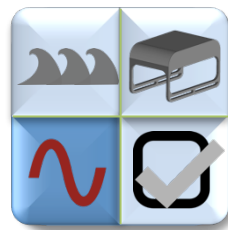
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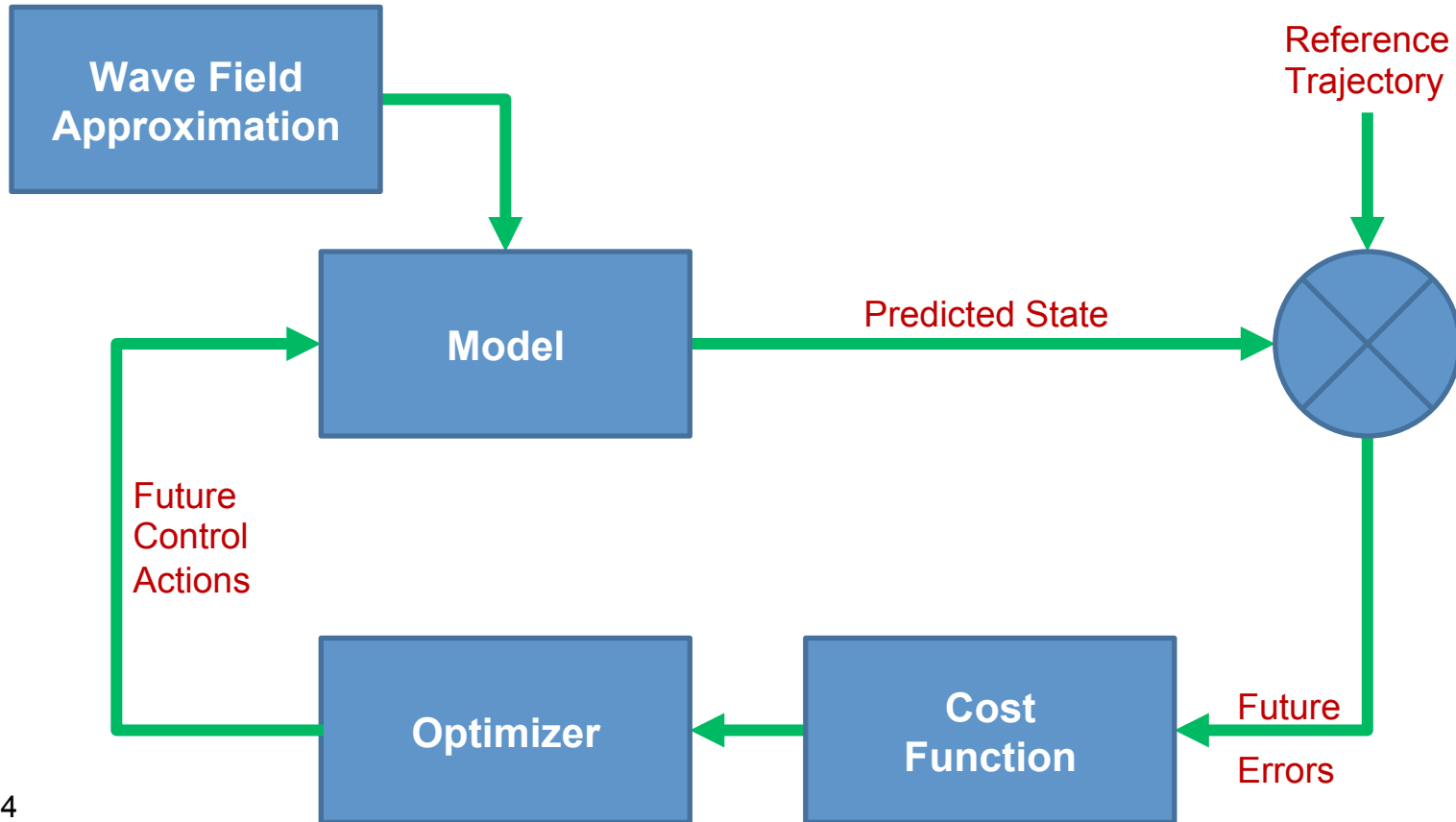


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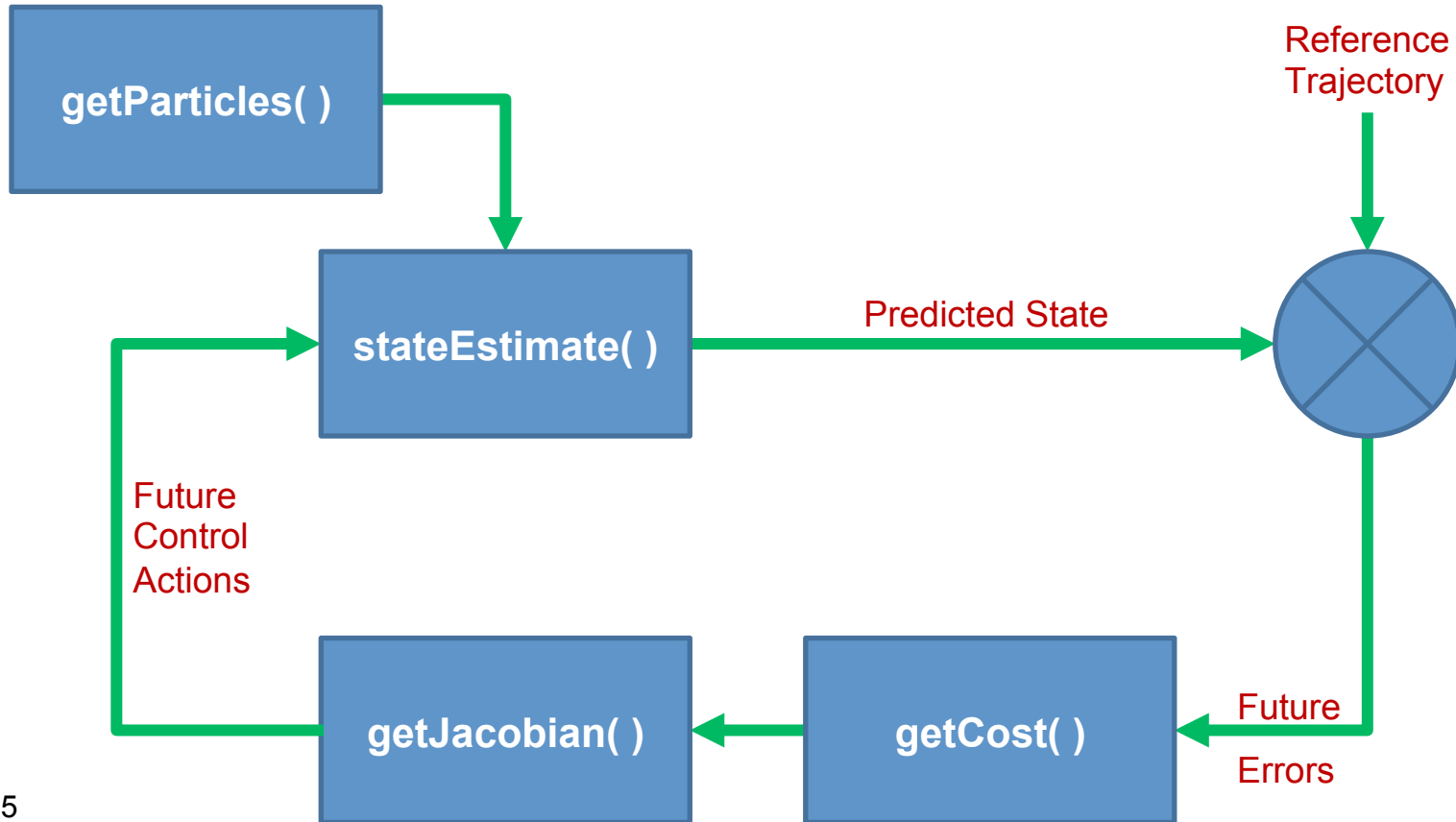
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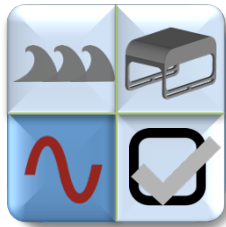
Algorithm



Algorithm

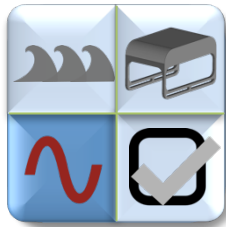


```
13:  while  $i < \text{maxIterations}$  and  $\delta > \text{exitCriteria}$  do
14:       $u_{i+1} \leftarrow u_i - \delta$ 
15:      for  $k \in [1, 2, \dots, N]$  do
16:           $\mathbf{v}_p, \dot{\mathbf{v}}_p \leftarrow \text{GETPARTICLES}(t, \Upsilon_i(k), \lambda)$ 
17:           $\Lambda \leftarrow \mathbf{v}_p, \dot{\mathbf{v}}_p$ 
18:           $\Upsilon_{i+1}(k) \leftarrow \text{STATEESTIMATE}(t, \text{robot}, \Lambda, u_{i+1}(k))$ 
19:           $J_{i+1}(k) \leftarrow \text{GETCOST}(\Upsilon_{\text{target}}, \Upsilon_{i+1}(k))$ 
20:       $\delta \leftarrow \text{GETJACOBIAN}(J_i, J_{i+1}, u_i, u_{i+1})$ 
21:       $u_i \leftarrow u_{i+1}$ 
22:       $J_i \leftarrow J_{i+1}$ 
23:       $\Upsilon_i \leftarrow \Upsilon_{i+1}$ 
24:       $i \leftarrow i + 1$ 
2 56  return  $u_{i+1}$ 
```



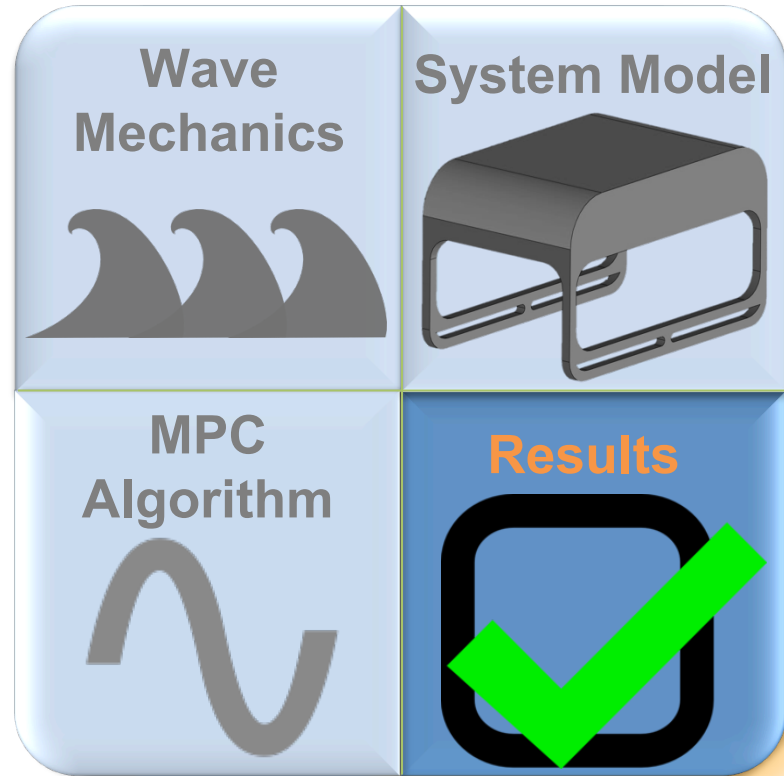
Algorithm

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Simulator (MPC Robot)



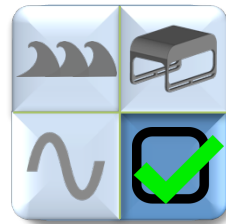


- Determine best performing horizon
- MPC performance versus PD control
- Resistance to noisy sensor observations

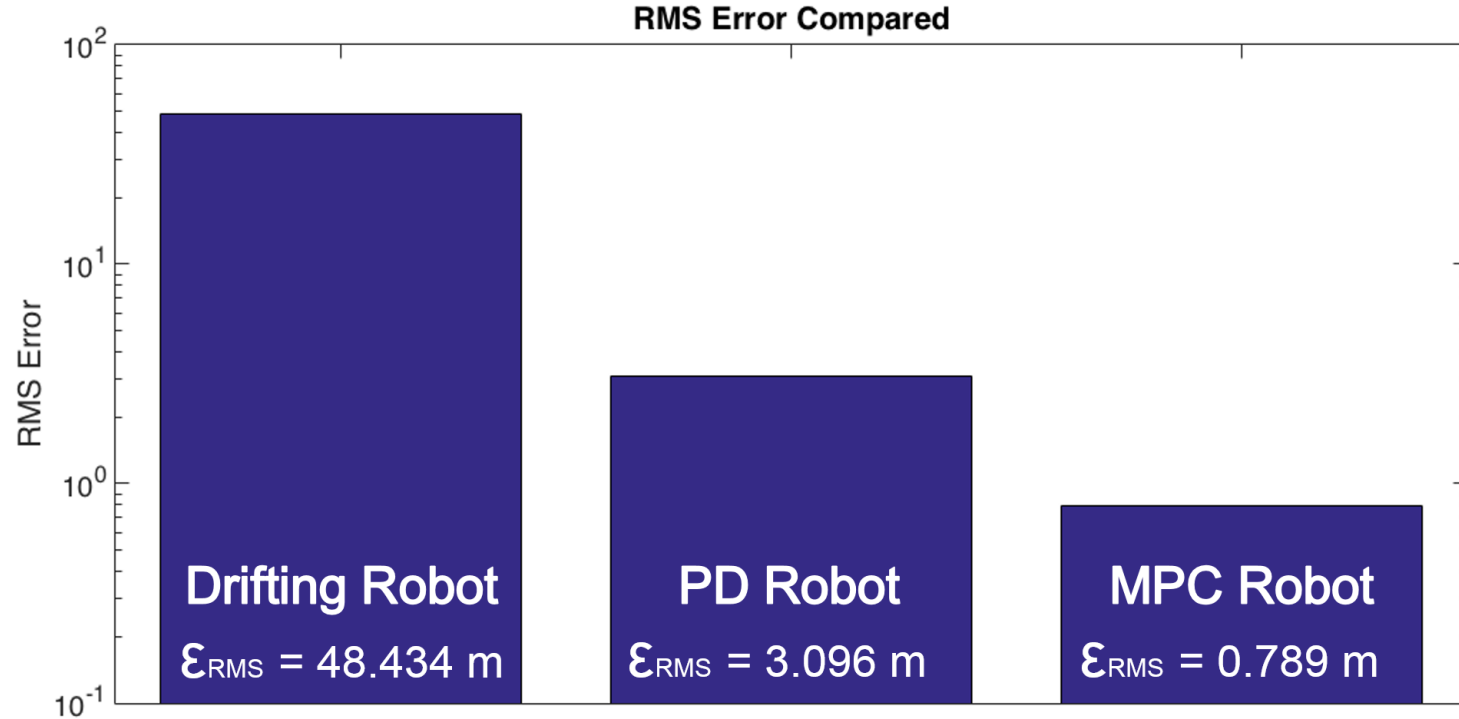


Horizon, s	0.2	0.4	0.8	1.0	1.6	2.0	3.0
$\epsilon_{RMS}, \text{ m}$	5.02	2.11	0.79	0.65	0.29	0.05	$9.0E-6$
$\Sigma t_{Calc}, \text{ s}$	1658	507.1	93.8	233.2	7808	21886	101252
$\bar{t}_{Calc}, \text{ s}$	6.90	2.11	0.39	0.97	32.53	91.19	421.88

- 0.8 s best balance of low error and time
- Poor performers on low & high end
- Total time: 240 s discretized by 0.2 s



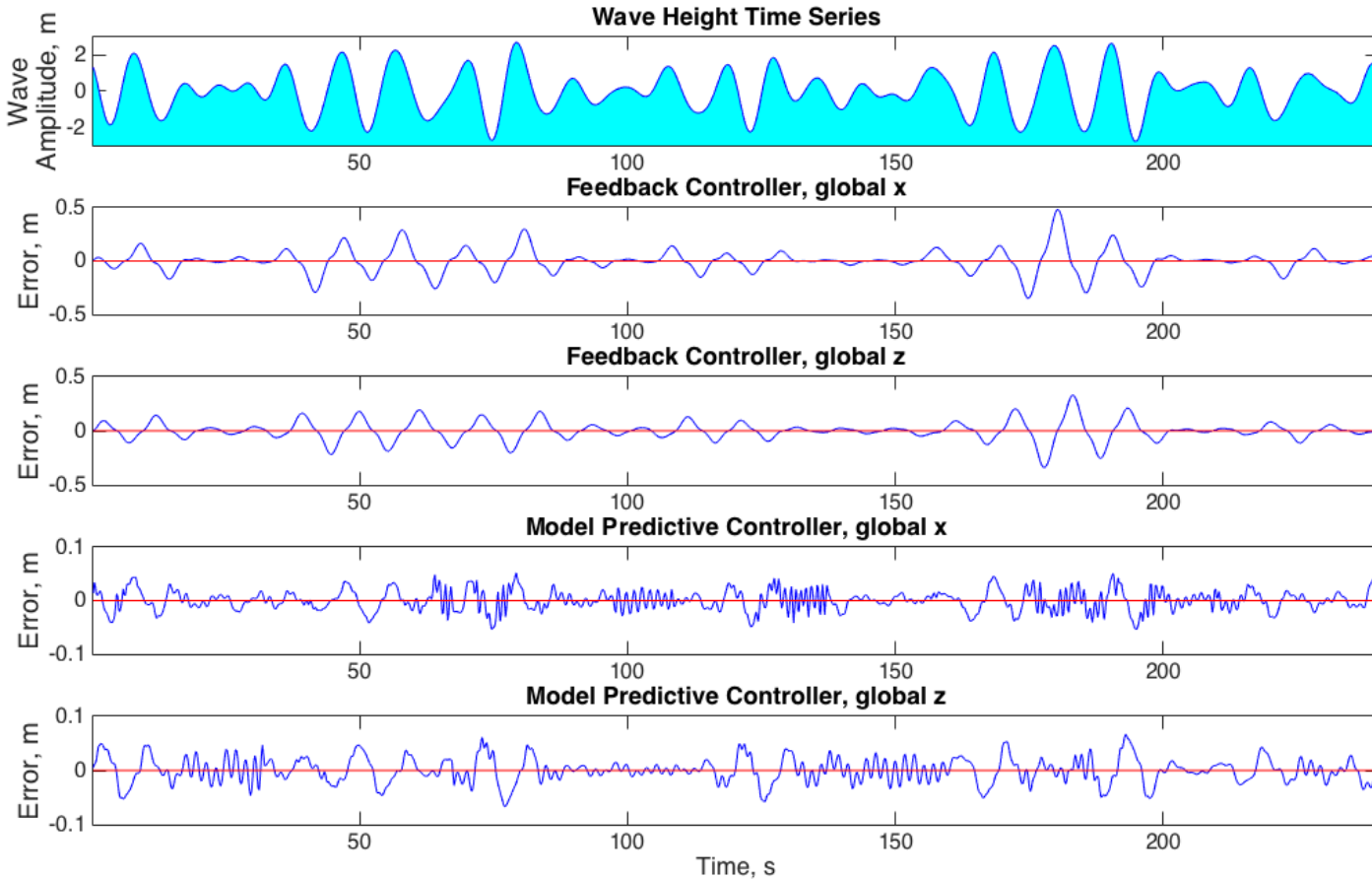
MPC Performance



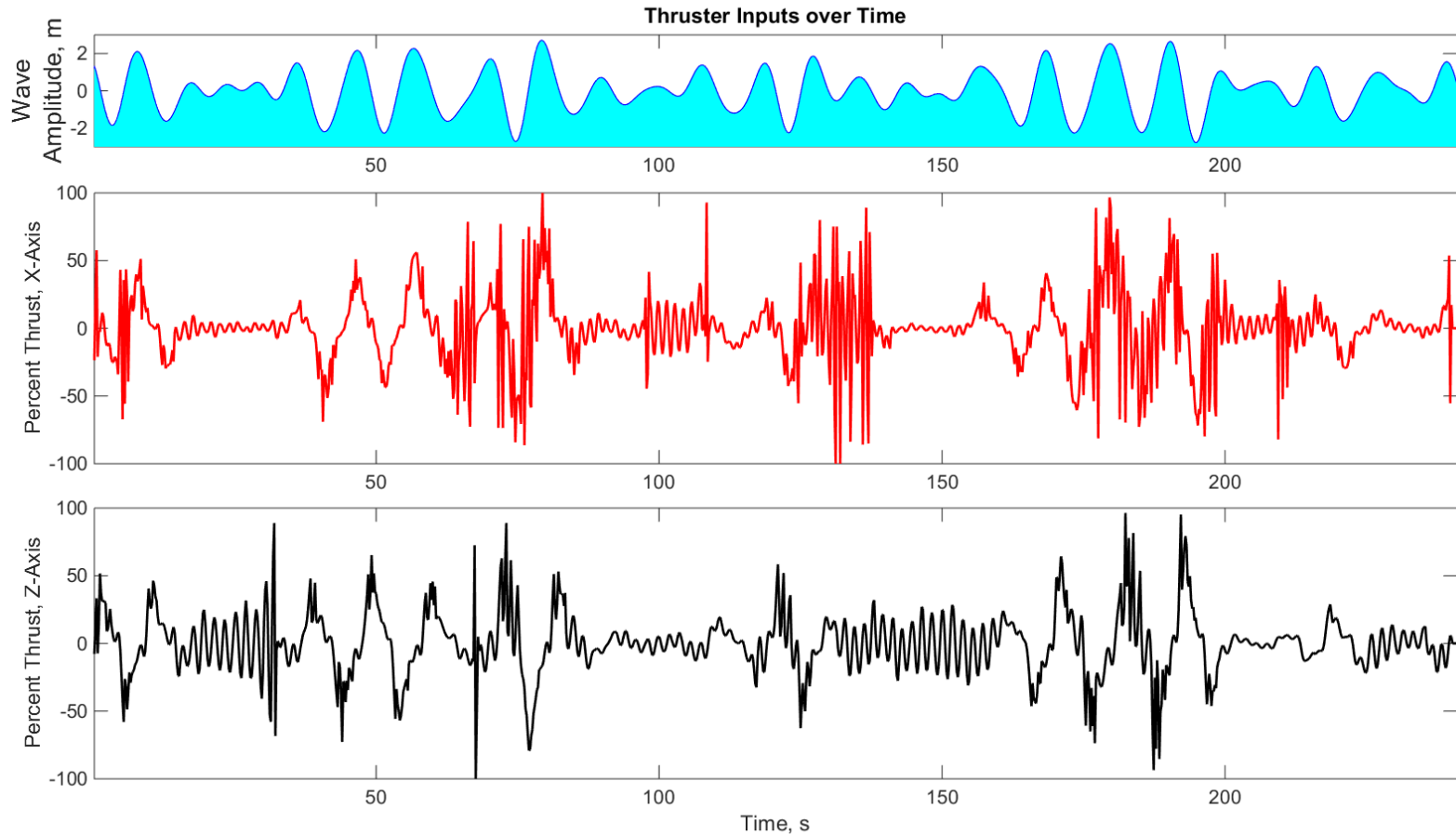
- 74% reduction in position error over PD



MPC Performance



MPC Performance

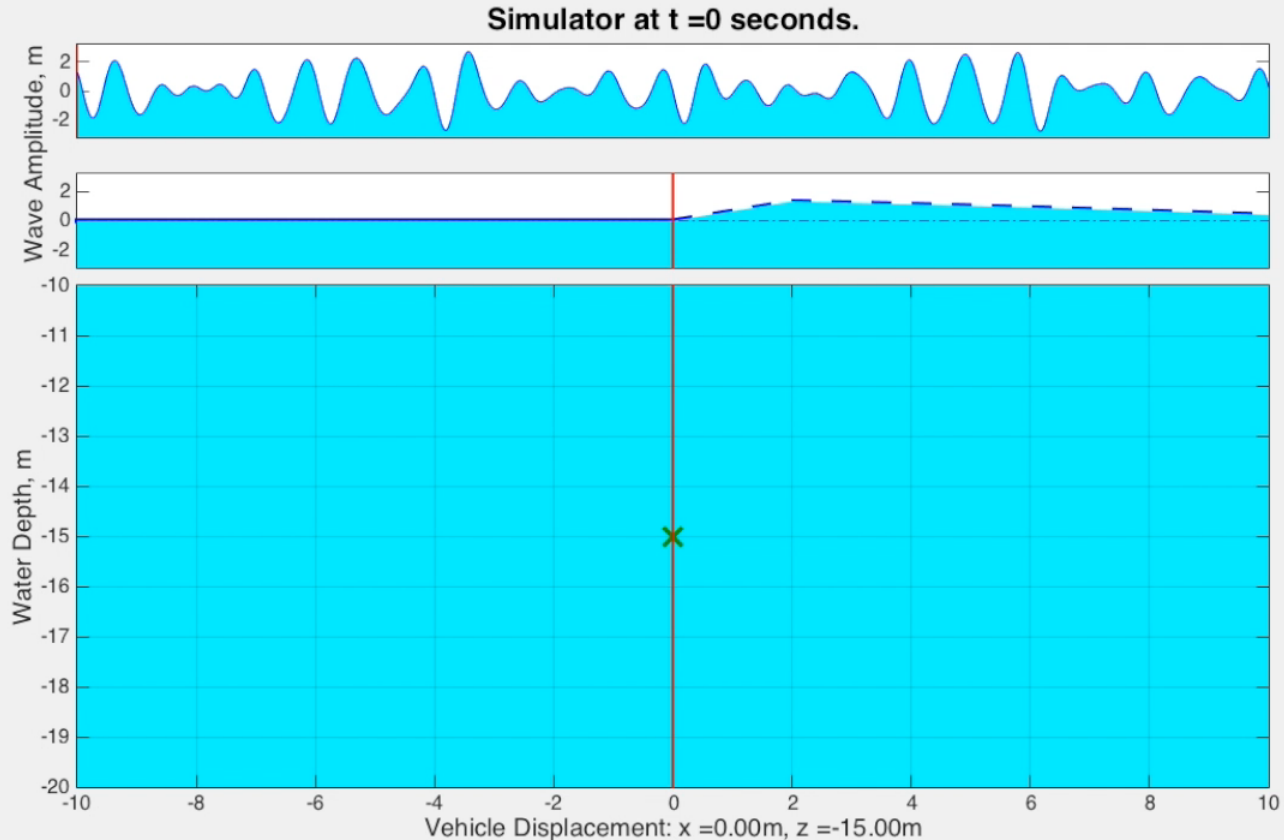


Impact of Gaussian Noise

- Observations of perceived wave state
- H term assigned maximum variance
- Minimal localization noise assumed
 - Deterministic PD case



Simulator (MPC w/ noise)



Impact of Gaussian Noise

- 50 simulations
 - *getForecast()* gets new noisy wave field at n^{th} step
- 44% reduction over PD
 - $\epsilon \downarrow RMS = 1.737m$
 - $\sigma = 0.059$
- Notable run time increase



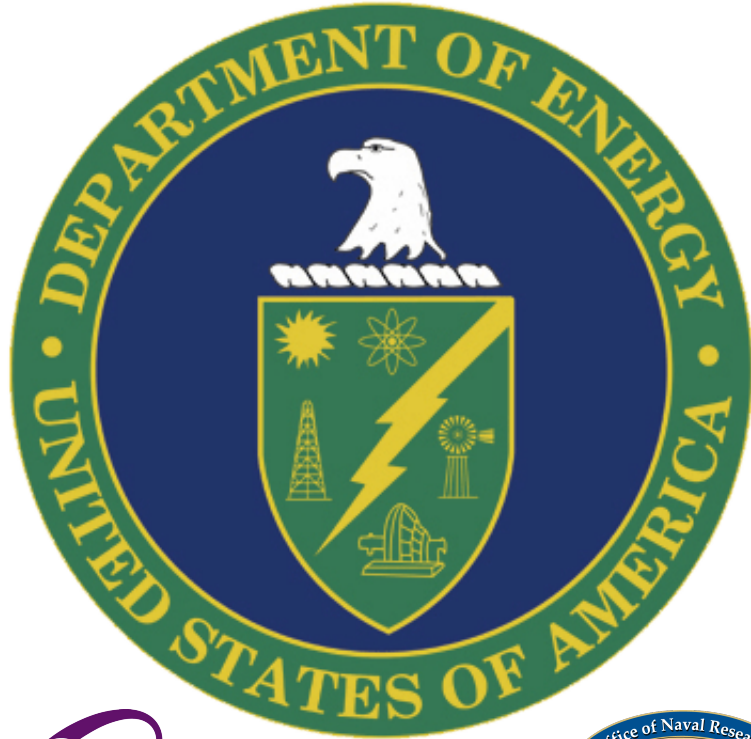
Summary of Contributions

- A feedforward control (MPC) method that can **forecast** and **compensate** for impending wave forces
- Application of the MPC algorithm to a simulated station-keeping robot
- Comparison of the MPC algorithm against traditional feedback (PD) control
- Algorithm resistance to noisy sensor observations of wave field parameters.
- Recommendations for choosing a prediction horizon

- Real-time wave prediction methods
- Neuro-Evolutionary control methods
 - Hydrodynamic simulation software packages
- System dynamics expanded
- More efficient optimization

Sponsors and Affiliations

Oregon State
UNIVERSITY



Questions?

