Squared Error Distortion Metrics for Motion Planning in Robotic Sensor Networks

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***Abstract*—We examine the problem of planning the trajectory of a robotic vehicle to gather data from a deployment of stationary sensors monitoring a set of dynamic source signals. The robotic vehicle and the sensors are equipped with wireless modems (e.g., radio in terrestrial environments or acoustic in underwater environments), which provide noisy communication across limited distances. In such scenarios, the robotic vehicle can improve its efficiency by planning an informed data gathering trajectory. We propose a novel performance metric for data gathering in robotic sensor networks based on the concept of squared error distortion. We analyze the formal properties of the distortion function, and we propose a sampling-based motion planning algorithm for optimizing data gathering tours for minimal distortion. The proposed algorithms are compared in simulation, and the results show that distortion metrics provide substantial improvements in data gathering efficiency.**

I. Introduction

The accurate measurement and interpolation of large-scale spatio-temporal processes is becoming increasingly important for sciences such as biology, climatology, geology, and oceanography. In terrestrial environments, phenomena of interest include seismic activity, volcanic activity, and catastrophic weather patterns. In marine environments, harmful algal blooms, oil spills, and other oceanographic events are extremely challenging to monitor effectively with available technology (e.g., satellites, drifters, and human-operated surface craft). Recent advances in autonomous robotic vehicles and sensor networks have made it feasible to study and predict these phenomena across large spatial scales and long periods of time, but a number of challenges still remain. For example, many currently deployed sensors must be removed from the field to download their data. The ability to gather sensor data in situ would improve the cost-effectiveness and lifespan of the sensors and would make such deployments feasible across larger scales.

In this paper, we propose a novel metric for coordinating the actions of a robotic vehicle collecting data from a stationary sensor network. The objective of the robotic vehicle is to process the collected data in order to monitor a signal from a dynamic source. The goal is to generate a good reconstruction of the source signal at the receiver for the purpose of monitoring. This problem setting becomes inherently estimation theoretic in nature, and hence we propose to measure the fidelity of the source estimate using the squared error *distortion* along the trajectory of the vehicle. The squared error distortion captures the average error over time in estimating the stochastic source. The distortion metric is widely used in the wireless communications literature (see [1]–[3] for details) to study the problem of lossy reconstruction of source sequences, but to our knowledge is unknown in the robotics literature.

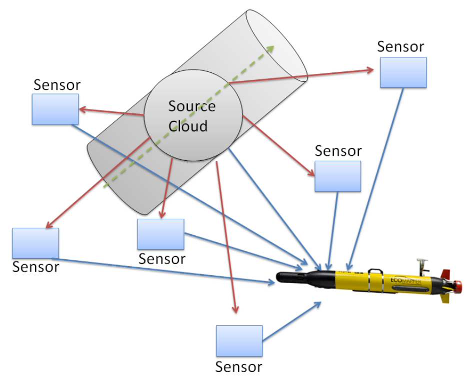

Fig. 1. Visualization of an autonomous vehicle gathering data from a deployment of sensors monitoring a moving source. We propose a novel metric based on squared error distortion that optimizes the trajectory of the vehicle to provide an accurate reconstruction of the signal emitted by the source.

The novelties of this paper include the introduction of a principled metric for autonomous data gathering based on squared error distortion, the formulation of motion planning algorithms to optimize this metric efficiently, and an empirical comparison of the proposed techniques.

II. Related Work

There has been increasing interest in the robotics community on the problem of coordinating robotic data mules for data collection tasks [4]. Recent research has often focused on ground robots constrained to download data from all deployed sensors. If the network is sparse, it can be efficient to partition the sensors into sub-networks and optimize the collection over each sub-network [5]. For denser networks, if the communication range of the sensors can be modeled as a fixed radius, it is possible to develop efficient motion planning methods based on the Traveling Salesperson (TSP) with Neighborhoods [6].

Such techniques have been implemented on robots operating in real-world environments showing the feasibility of robotic data mules with current technology [7].

For many applications of robotic sensor networks, a fixed communication radius is not a valid assumption due to gradual degradation of packet error rate over distance [8]. To provide more realistic communication modeling, two-ring communication models have been explored [9], and methods that optimize based on expected network latency have been proposed [10]. Similar approaches have also been applied to improve the placement of sensors to maximize communication efficiency [11].

Robotic data collection from sensor networks has also been applied to applications in underwater domains. In such domains, communication is limited to long-range and low-bandwidth acoustic communication [12] or shorter-range and higher-bandwidth optical communication [13]. In our prior work, we integrated robotic motion planning techniques with realistic acoustic communication modeling for the problems of station-keeping [14] and underwater search [15]. In recent work, we explored the problem of robotic data collection in underwater sensor networks, and we proposed motion planning algorithms that optimize assuming a probabilistic communication model [16]. In the current paper, we introduce a new metric for such tasks that measures the integrity of sensed information that is communicated over a channel.

III. Problem Formulation

In this section, we formulate the problem of mobile data collection from stationary sensors using an autonomous vehicle. We consider a pre-deployed network of K sensors located in $\mathbb{R}^d$ with $d \in \{2,3\}$, which yields the 2D and 3D problems respectively. We assume that the location $l_s(k) \in \mathbb{R}^d$ is given for each sensor $k \in [1:K]$, where K is the total number of deployed sensors.

In the context of gathering data from pre-deployed sensor fields, the motion planning optimization problem is to generate a trajectory for an autonomous vehicle that retrieves data from the sensors and minimizes the traversal cost of the trajectory. The autonomous vehicle moves along a trajectory $\mathcal{P} = [l_v(1), \cdots, l_v(T)]$ (a trajectory is represented as a collection of points in $\mathbb{R}^d$) to gather data.

The moving vehicle can be thought of as a receiver that processes the received output from the sensors. The location $l_v \in \mathbb{R}^d$ of the vehicle is assumed to be known with reasonable fidelity (e.g., using an onboard localization system). The movement of the vehicle is controlled and may be subject to constraints, such as obstacles or vehicle kinematics. Based on these constraints, a traversal cost $C(l_1, l_2)$ is defined for all pairs of points $l_1, l_2 \in \mathbb{R}^d$.

The mobile data collection path planning problem requires the optimization of a data quality objective function given constraints on budget (e.g., time, fuel, or energy). We propose a novel objective function that minimizes the squared error distortion of a set of dynamic source signals. This metric will be applicable across a wide range of robotic monitoring domains.

A. Distortion Metrics

We examine the scenario where a robotic vehicle must gather data from a deployment of stationary sensors to estimate a set of correlated dynamic sources. The dynamic sources $\{S^{(m)}\}_{m=1}^{M}$ are assumed to be located at $l^{(m)} \in \mathbb{R}^d$, $\forall\, 1 \leq m \leq M$. The sources can be modeled as a discrete time stochastic process $\{S_i^{(m)}\}_{i\geq 1}$, $\forall\, 1 \leq m \leq M$. The stochastic process $\{S_i^{(m)}\}_{i\geq 1}$ for each $m \in [1:M]$ is assumed to be i.i.d. over time, whereas the different sources at any instant of time i can be arbitrarily correlated to each other, or in other words, $\{S_i^{(m)}\}_{m=1}^{M}$ are jointly distributed random variables with joint density function $f_M(s_i^{(1)}, \cdots, s_i^{(M)})$ for each $i \geq 1$. For the purpose of this work, we will assume that $\{S_i^{(m)}\}_{m=1}^{M}$ is a zero mean jointly Gaussian random variable with a given covariance matrix, i.e., $(S_i^{(1)}, \cdots, S_i^{(M)})^T \sim \mathrm{N}(0, \Sigma_S)$. Note that when the covariance matrix Σ_S is diagonal, then the sources are independent of each other.

Each sensor observes the sources through a noisy broadcast channel $p(s_1, \ldots, s_K|s)$. The observation of each sensor k, which we refer to as the state of the sensor k, can be modeled as a discrete time stochastic process $\{S_{ki}\}_{i\geq 1}$. The stochastic process $\{S_{ki}\}_{i\geq 1}$ for each $k \in [1:K]$ is assumed to be i.i.d. over time, whereas the state of different sensors at any instant of time i can be arbitrarily correlated to each other, or in other words, $\{S_{ki}\}_{k=1}^{K}$ are jointly distributed random variables with joint density function $f_K(s_{1i}, \cdots, s_{Ki})$ for each $i \geq 1$. For the purpose of this work, we will assume that $\{S_{ki}\}_{k=1}^{K}$ is a zero mean jointly Gaussian random variable with a given covariance matrix, i.e., $(S_{1i}, \cdots, S_{Ki})^T \sim \mathrm{N}(0, \Sigma)$. The covariance matrix can be evaluated by assuming an additive white Gaussian noise channel between the source and each of the sensors, which is given by

$$S_{ki} = \sum_{m=1}^{M} h_m(L(l^{(m)}, l_s(k))) S_i^{(m)} + Z_{ki},\ \forall\, 1 \leq k \leq K, \tag{1}$$

where the receiver noise at the k-th sensor is $Z_{ki} \sim \mathrm{N}(0,1)$, and $h_m(\cdot)$ is channel coefficient, which is some deterministic function of the distance $L(\cdot)$ between the source m and the sensor k. Note that the covariance matrix is time invariant since the process is i.i.d.

Each of the stationary sensors k is capable of transmitting a function of its observation $X_{ki} = f(S_k^i)$ (note that X_{ki} is a causal function of the sensor state S_k) to the vehicle through a communication channel, which is not only influenced by the receiver noise, but also by the presence of the stochastic source in the medium. We assume an expected average transmission power constraint at the sensors such that

$$\sum_{i=1}^{n} \mathsf{E}(x_{ki}^2(S_k^i)) \leq nP_k,\ \forall\, 1 \leq k \leq K, \tag{2}$$

where the expectation is over the random source sequence $S^{(m)}$. The communication channel between the sensors and the vehicle is modeled as a noisy state dependent Gaussian multiple access channel $p_t(y|s_1,\ldots,s_K)$ and its ouput is given by

$$Y_i = \sum_{k=1}^{K} h_k(L(l_v(t+1),l_s(k)))X_{ki}(S_k^i) + \sum_{m=1}^{M} S_i^{(m)} + Z_i, \quad (3)$$

where the receiver noise $Z_i \sim \mathrm{N}(0,1)$ and $\{h_k\}_{k=1}^K$ are channel coefficients, which are again functions of the distance between the vehicle and corresponding sensor. Fading coefficients have been omitted from the second term to simplify the following equations.

The vehicle's goal is to move along a trajectory collecting data from the sensors to estimate the underlying sensor field with maximum fidelity. The fidelity of a source estimate for each of the sources at a particular location is measured by the *expected distortion*

$$\begin{aligned} D(l_v)^{(m)} &= \mathsf{E}(d(S^{(m)n},\hat{S}^{(m)n})) \\ &= \frac{1}{n}\sum_{i=1}^{n} \mathsf{E}(d(S_i^{(m)},\hat{S}_i^{(m)}(Y^m))),\ \forall\, 1 \le m \le N, \end{aligned} \quad (4)$$

where $d : \mathcal{S} \times \hat{\mathcal{S}} \to [0,\infty)$ is a distortion measure between a state symbol $s \in \mathcal{S}$ and a reconstruction symbol $\hat{s} \in \hat{\mathcal{S}}$, which is a function of the observation y^n at a particular location l_v. In this work, we will consider squared error distortion $d(s,\hat{s}) = (s-\hat{s})^2$. The aim of the autonomous vehicle is to estimate the underlying m-th source $S^{(m)n}$ in minimum mean squared error (MMSE), i.e.,

$$D(l_v)^{(m)} = \min_{f_t(\cdot),\hat{S}_t^{(m)}(\cdot)} \frac{1}{n}\sum_{i=1}^{n} \mathsf{E}(S_i^{(m)} - \hat{S}_i^{(m)}(Y^n))^2, \quad (5)$$

where $f_t(\cdot)$ is the encoding function at the sensors. The estimate is made at each location and updated along the trajectory as the vehicle gathers information.

As the vehicle receives more information from the sensors along its trajectory of travel, it updates the effective distortion $D_e(\mathcal{P}^{t+1})^{(m)}$ in the estimation of each of the underlying sources m, which is given by

$$D_e(\mathcal{P}^{t+1})^{(m)} = \frac{tD_e(\mathcal{P}^t)^{(m)} + D(l_v(t+1))^{(m)}}{t+1}, \quad (6)$$

where $D_e(\mathcal{P}^1)^{(m)} = D(l_v(1))^{(m)}$, and $D(l_v(t))^{(m)}$ is given by (4). In some scenarios, it may be desirable to give recent distortion values additional weight in the total effective distortion. This can be achieved by adjusting the weighting of $D_e(\mathcal{P}^t)^{(m)}$ versus $D(l_v(t+1))^{(m)}$ in (6).

By using the function $D_e(\cdot)$ as the measure of data quality, we now have a fully defined robotic data collection problem.

Problem 1: Given a trajectory cost function $C(\mathcal{P})$, and a set of possible trajectories $\mathcal{P} \in \psi$, find

$$\begin{aligned} \mathcal{P}^* &= \operatorname*{argmin}_{\mathcal{P}\in\Psi} \sum_{m=1}^{M} \beta_m D(\mathcal{P})^{(m)} \\ &\text{s.t. } \beta_m \ge 0,\ \sum_{m=1}^{M}\beta_m = 1 \text{ and } C(\mathcal{P}) \le B, \end{aligned} \quad (7)$$

where $D(\mathcal{P})^{(m)}$ is the distortion function, β_m are pre-established weights that signify the relative importance of the different sources, T is the index of the last point on the trajectory and B is a budget threshold on the cost of the trajectory (e.g., maximum mission time, battery life, or remaining fuel).

B. Communication Strategy

In this subsection, we propose an encoding strategy at the sensors and a decoding strategy at the vehicle that minimizes the one-step distortion $D(l_v(t+1))^{(m)}$ in estimating the source m for a particular vehicle location $l_v(t+1)$ at time $t+1$. We assume that the stationary sensors have limited capabilities, and hence we choose the encoding function $X_{Ki} = f(S_k^i) = \alpha_k S_{Ki}$, where $\alpha_k = \sqrt{P_k/\Sigma(k,k)}$ is a constant chosen to satisfy the input power constraint of P_k at sensor k. This simple amplification strategy at the sensors may be suboptimal, but with the practical constraint of limited processing power at the sensors, amplify-and-forward is the most natural coding strategy to consider and offers good performance as seen in the sequel.

Let us compute the one-step distortion $D(l_v(t+1))^{(m)}$ in estimating the m-th source for the proposed encoder and decoder as the data gathering vehicle moves from position $l_v(t)$ to $l_v(t+1)$. We assume that when the vehicle is at location $l_v(t)$, the effective distortion in estimating the source m till time t is given by $D_e(\mathcal{P}^t)^{(m)}$. To compute the one-step distortion, let us look at the the received output at the moving vehicle, when it is at location $l_v(t+1)$. It is given by

$$Y_i = \sum_{k=1}^{K} h_k(L(l_v(t+1),l_s(k)))\alpha_k S_{ki} + \sum_{m=1}^{M} S_i^{(m)} + Z_i \quad (8)$$

$$= [\ S_{1i}\ \ \ldots\ \ S_{Ki}\ ]\underline{h} + [\ S_i^{(1)}\ \ \ldots\ \ S_i^{(M)}\ ]\underline{1} + Z_i, \quad (9)$$

where $\underline{h}$ and $\underline{1}$ are respectively $K$and M dimensional column vectors with their j-th component given by $\underline{h}_j = h_j(L(l_v\,(t+1)\,, l_s\,(1)))\alpha_j$ and $\underline{1}_j = 1$. We choose $\hat{S}_i^{(m)}(y_i) = \mathsf{E}(S_i^{(m)}|Y_i = y_i) = \frac{\mathsf{E}(S^{(m)}Y)}{\mathsf{E}(Y^2)} y_i$. The expected distortion in estimating the m-th source at the vehicle, when it is at location $l_v(t+1)$ is

$$D(l_v(t+1))^{(m)} = \sigma^2 - \frac{\mathsf{E}(S^{(m)}Y)^2}{\mathsf{E}(Y^2)}, \quad (10)$$

where

$$\frac{\mathsf{E}(S^{(m)}Y)^2}{\mathsf{E}(Y^2)} = \frac{(\Sigma_c(m,:)\underline{h} + \Sigma_S(m,:)\underline{1})^2}{\underline{h}^T\Sigma\underline{h} + 2\underline{h}^T\Sigma_c^T\underline{1} + \underline{1}^T\Sigma_S\underline{1} + 1}. \quad (11)$$

Here Σ_c is the $M \times K$ cross-correlation matrix between the sources and the sensor observations defined earlier. The one-step distortion $D(l_v(t+1))^{(m)}$ is a function of the distance between the source and the sensors, the sensors and the vehicle, and the covariance matrix of the source Σ_S. So to calculate this distortion function, the vehicle requires the knowledge of these parameters. We will relax these assumptions in the next section.

IV. Moving Sources

So far in our discussion, we have assumed full channel state information at the sensors and the vehicle, i.e., knowledge of $\{h_m\}_{m=1}^M$ in (1) and $\{h_k\}_{k=1}^K$ in (3). This assumption may not be realistic when the dynamic source is not fixed at a particular location. In this section, we extend our framework to include the scenario of monitoring a moving source.

The sensors and the vehicle attempt to estimate the exact location of the source at each instant of time. We assume the vehicle knows the position of the source within an uncertainty region $\mathcal{S}$ (the position of the source is distributed according to $p(l), \;\; l \in \mathcal{S}$) at each instant of time. Such an estimate could be achieved through an extended Kalman filter (EKF) or other tracking method. We also assume that in the moving source case, the uncertainty region moves across the sensor field according to the dynamics of the source motion (see Figure 1).

We use encoding and decoding strategy outlined below for the moving source case. Suppose that the exact location of the source at any instant of time is $l \in \mathcal{S}$. The sensors observe the source through a noisy communication channel, the output of which at sensor k is given by (1) with $M = 1$. The sensors again send a amplified version of their observations to the vehicle to satisfy the transmission power constraint of (2). The amplification factor

$$\alpha(k) = \sqrt{\frac{P_k}{\Sigma(k,k)}} = \sqrt{\frac{P_k}{h^2(L(l, l_s(k)))\sigma^2 + 1}} \qquad (12)$$

is a function of the channel coefficient, which in turn depends on the exact location of the source $l \in \mathbb{R}^d$. Since the input power constraint has to be met for all possible locations of the source inside the sphere $\mathcal{S}$, for a fixed P_k, each sensor finds a location on or within the source cloud which is the solution of the following optimization problem

$$\hat{l} = \underset{l \in \mathcal{S}}{\operatorname{argmax}}\, \mathsf{E}(S_k)^2 \equiv \underset{l \in \mathcal{S}}{\operatorname{argmax}}\, h^2(L(l, l_s(k))). \qquad (13)$$

The modified amplification factor at sensor k is determined by calculating $\alpha(k)$ assuming that the source is at $\hat{l}$. The value of $\alpha(k)$ derived in this fashion will always satisfy the input power constraint since for any source position $l \in \mathcal{S}$

$$\mathsf{E}(X_k)^2 = \mathsf{E}(\alpha_k S_k)^2 \qquad (14)$$

$$= \frac{P_k(h^2(L(l, l_s(k)))\sigma^2 + 1)}{h^2(L(\hat{l}, l_s(k)))\sigma^2 + 1} \leq P_k. \qquad (15)$$

With this encoding strategy, the vehicle performs path planning by evaluating the one-step distortion in estimating the source based on an MMSE estimator discussed in the previous sections. However, since the one-step distortion is a function of the exact location of the source (which is not known to the vehicle), the objective of the vehicle is to determine a value for the one-step distortion $D(l_v)$, such that it is close to the one-step distortion value with the true location of the source. Suppose that the one-step distortion function if the vehicle knows the exact location of the source is given by $D_l(l_v)$, then without the knowledge of the source location, the vehicle chooses a one-step distortion function $D(l_v)$ which is equal to the

$$D(l_v) = \int_{\mathcal{S}} D_l(l_v) p(l) dS = \mathsf{E}(D_l(l_v)), \qquad (16)$$

where $p(l)dS$ is the probability of the source being in an infinitesimal volume around $l \in \mathcal{S}$. To calculate $\mathsf{E}(D_l(l_v))$, the vehicle randomly selects n points $l(j), \; 1 \leq j \leq n$ from on or within the region $\mathcal{S}$ i.i.d. according to the distribution $p(\cdot)$. The one-step distortion $D(l_v)$ is then given by

$$D(l_v) = \frac{1}{n} \sum_{j=1}^{n} D_{l(j)}(l_v). \qquad (17)$$

This empirical mean of the distortion value converges to $\mathsf{E}(D_l(l_v))$ in the limit as the number of sample points $n \to \infty$ by the law of large numbers. It is easy to see that the decoding strategy employed is optimal with respect to the amplify-and-forward sensor strategy since the channel coefficient is averaged over all possible realizations inside the uncertainty cloud.

V. Properties of the Distortion Metric

In this section, we show that the distortion metric defined in the previous section is neither monotonic nor submodular, two properties often associated with information optimization objectives [17]. These formal properties will provide insight into the design of motion planning algorithms suitable for optimizing data gathering trajectories that minimize distortion.

Definition 1: Let Ω be the set of grid points of the sensor field. A function $f : 2^\Omega \to \mathbb{R}$ defined on the subsets of Ω, is said to be *monotonically decreasing* if for every $T \subseteq S \subseteq \Omega$, we have that $f(T) \geq f(S)$.

Definition 2: A function $f(\cdot)$ defined above, is said to be *submodular* iff for every $X \subseteq Y \subseteq \Omega$ and $x \in \Omega \setminus Y$ we have that $f(X \cup \{x\}) - f(X) \geq f(Y \cup \{x\}) - f(Y)$.

It is easy to see that the effective distortion function $D_e(\cdot)$ is *not monotonically decreasing* since it is inversely proportional to the channel qualities between the vehicle and the sensors (and thus proportional to the Euclidean distance between the sensors and the vehicle according to our channel model). Hence, along an arbitrarily chosen trajectory $\mathcal{P}$, the effective distortion may increase or decrease depending on the varying channel qualities along the trajectory.

The effective distortion metric $D_e(\cdot)$ is also *not submodular*. To see this, let us consider a trajectory $\mathcal{P}_1 \subseteq \mathcal{P}_2$ and consider a new point $\{x\} \in \Omega \setminus \mathcal{P}_2$. The effective distortions along the trajectory are given by:

$$D_e(\mathcal{P}_1 \cup \{x\}) - D_e(\mathcal{P}_1) = \frac{D(x) - D_e(\mathcal{P}_1)}{|\mathcal{P}_1| + 1} \quad (18)$$

$$D_e(\mathcal{P}_2 \cup \{x\}) - D_e(\mathcal{P}_2) = \frac{D(x) - D_e(\mathcal{P}_2)}{|\mathcal{P}_2| + 1}, \quad (19)$$

where $D(x)$ is the one-step distortion of the source observation S_i at location $x \in \Omega$. Since the effective distortion of the source observations $D_e(\cdot)$ is not monotonic (varies randomly along a trajectory depending on the variation of channel quality along the path), there is no definite ordering between $D_e(\mathcal{P}_1 \cup \{x\}) - D_e(\mathcal{P}_1)$ and $D_e(\mathcal{P}_2 \cup \{x\}) - D_e(\mathcal{P}_2)$ for any selected paths in the sensor field. Thus the effective distortion metric $D_e(\cdot)$ is also not submodular.

VI. Motion Planning Algorithms

We now discuss a sampling-based motion planning algorithm that efficiently generates trajectories to minimize the distortion metric while also maintaining the cost budget constraints. As discussed above, the distortion metric is neither monotone nor submodular, which excludes motion planning algorithms that rely on these assumptions [18], [19]. The distortion metric is also not convex, and it often contains a number of local minima. Thus, gradient-based methods are likely to perform poorly (see the simulations in the following section).

Our approach extends the RRT* [20] and Information-Rich RRT [21] algorithms to provide optimized distortion minimization. The key idea is to sample the configuration space of the vehicle (i.e., locations where the vehicle may visit) and to build up a tree of possible trajectories by incrementally extending candidate trajectories towards the sampled points. The main challenges presented by the distortion metric are (1) calculating the distortion at each node on the tree in an efficient manner, and (2) focusing the tree generation such that candidate paths satisfy the budget requirements. We employ the Rapidly-exploring Information Gathering (RIG) algorithm proposed in our prior work, which is designed to address these challenges [22].

One desirable property of the distortion metric is that the distortion at time $t+1$ is fully defined by the next segment of the vehicle's trajectory, the locations of the sensors, the location of the information source, and the source distortions $D_e(\mathcal{P}^t)^{(m)}$ at time t along that trajectory. It is straightforward to build trajectories in an incremental fashion by storing the trajectory segments and the matrix $D_e(\mathcal{P}^t)^{(m)}$ at each node. In the case of an unknown source, it is sufficient to store an estimate of the distortion and then propagate that estimate forward. Through this incremental path generation, we can extend the RIG algorithm to the budget constrained distortion minimization problem. We will refer to the extended algorithm as the BCDM-RRT.

VII. Simulations

We now provide simulations to test the proposed motion planning techniques and their effectiveness in minimizing the distortion metric. The simulations were performed in C++ on an Ubuntu Linux desktop with a 3.2 GHz Intel i7 processor with 9 GB of RAM. We first examine the performance of the proposed BCDM-RRT sampling-based motion planner in a 10 km $\times$ 10 km 2D environment with 10 randomly placed sensors and a randomly placed source. The vehicle is capable of unconstrained motion with a maximum speed of 1 km/hr. The environment contains a varying number of circular obstacles generated with random radii up to 5 km. In these simulations, the cost constraint considered is the mission time, which represents the time that vehicle may remain deployed.

We compare the BDM-RRT method to a gradient-based approach. Gradient-based optimization methods have previously been used in mobile sensor networks to optimize for localization accuracy [23]. We also compare to a heuristic that moves directly to the source, which has been applied in prior work for robotic data muling [7].

A. Stationary Sources

Figure 2 shows the results from data gathering tours using 1000 random deployments with an increasing number of obstacles for two mission times. Examples are shown for the case of a single source and for the case of multiple (five) sources. In all cases, the BCDM-RRT outperforms the gradient-based method due to its ability to escape local-minima in the distortion function and find a more globally optimal path. The advantage of the BCDM-RRT is greater with increasing mission times and with fewer obstacles (due to fewer constraints on the vehicle's motion). The BCDM-RRT was run with 100,000 samples, which took approximately 10 seconds per deployment.

The benefit of BCDM-RRT over the gradient-based method is also significant in the multi-source case where there is increased variation of the objective function caused by the presence of multiple sources. We also compare to two heuristics that are unaware of the underlying distortion: a random walk and a strategy that moves directly to the information source. The BCDM-RRT and gradient-based methods both outperform these heuristics, which demonstrates the importance of considering distortion in the trajectory optimization.

B. Moving Sources

We also examine the benefit of utilizing the distortion metrics for the case of a single moving source. Figure 2 shows results from simulations using a source that moves on a fixed trajectory. The trajectory of the source is known to the vehicle, but the exact location is only known within 1 km (i.e., the uncertainty region is a sphere with radius 1 km).

In these simulations, since the location of the source is not known exactly, the heuristic strategy moves to the center of the uncertainty region. Similar to the case of stationary sources, the BCDM-RRT method (with 10,000 samples) and the distortion gradient method outperform the heuristic methods. The gradient-based method is more competitive here because the necessity of estimating the position of the source negates some of the benefit of long-term planning. Even in this

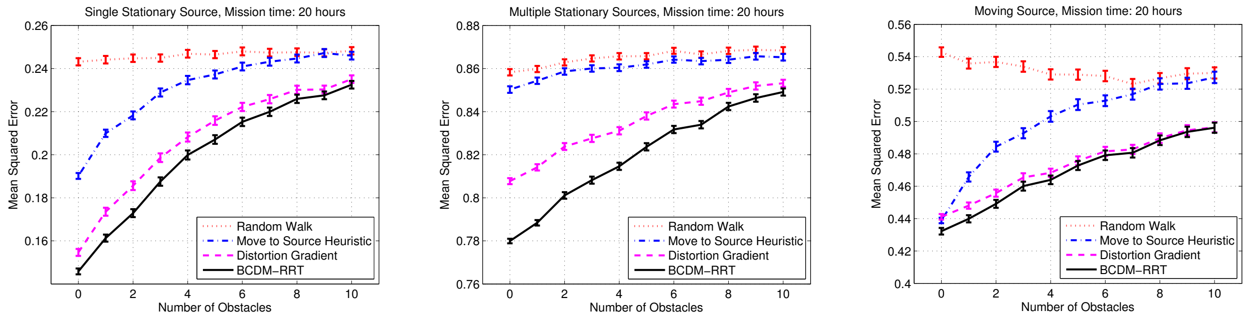

Fig. 2. Comparison of the BCDM-RRT sampling-based motion planning algorithm to a gradient-based approach and two heuristic strategies in a 10 km × 10 km environment with obstacles. The simulated vehicle is capable of unconstrained motion at a maximum speed of 1 km/hr. Results are shown for a single source and for five sources. The proposed method provides improved estimation of the source signal for a given trajectory length. Each data point is averaged over 1000 random sensor deployments, and error bars are one SEM.

challenging scenario, the BCMD-RRT still provides improved performance.

VIII. Conclusions and Future Work

In this paper, we have examined the problem of cost-constrained motion planning of a robotic vehicle to gather data from a network of stationary sensors tracking a dynamic source. Since the underlying objective of path planning is to collect data from the sensors to estimate a stochastic source sequence, we proposed a performance metric based on the concept of minimizing the squared error distortion in the sensed signal. We analyzed the formal properties of the distortion function, proposed a communication strategy, and evaluated the distortion metric for this communication strategy. In addition, we extended our results to moving sources, which is of immense practical importance for many spatio-temporal monitoring applications. We introduced a sampling-based motion planning algorithm for optimizing data gathering tours for minimal distortion, and we showed that planning using distortion metrics provides significant improvements in data gathering efficiency versus naive methods.

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