Real-Time Stochastic Optimization for Energy-Efficient Trajectories

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***Abstract*—We present an iterative optimization algorithm for path planning. The algorithm samples smoothly deformed paths around a current best path and then updates the best path guess based upon a given cost function. We apply this algorithm to the problem of finding an energy-efficient path in an underwater environment. Results are shown for both a simulated current environment and using a Regional Ocean Modeling System (ROMS) ocean current data set. These results show that our algorithm is able to plan more feasible energy-efficient paths than current methods.**

I. Introduction

When operating in underwater environments, a number of disturbances can effect Autonomous Underwater Vehicle (AUV) operation. One such disturbance is ocean currents, which can either help or hinder the AUV from reaching its goal destination. Further, ocean current maps are quite coarse and potentially inaccurate, making fully offline path planning difficult, as blindly executing these plans can result in undesired and potentially dangerous behavior.

Previous work in energy efficient path planning has primarily looked in two directions. Both Witt and Dunbabin [6] and Subramani and Lermusiaux [5] looked at varied optimization methods; however their solutions are computationally expensive and are not suitable for online applications. Alternatively, Lee et al. [4] and Huynh et al. [1] use A* like search methods to find paths. However the necessary discretization does not allow the vehicle to fully utilize the currents and can produce paths with infeasible transitions.

In this work we present an algorithm, Energy Efficient Stochastic Trajectory Optimization (EESTO), that is capable of calculating energy efficient paths online. EESTO builds on previous stochastic optimization motion planners, such as STOMP presented by Kalakrishnan et al. [2], by removing the assumption that waypoints are equally spaced in time which is necessary to allow fully for variations in vehicle thrust and to plan energy efficient paths.

II. Algorithm

Following the notation of STOMP, our motion planning problem is defined as finding a path from a given starting location to a desired goal location. The trajectory, $\tilde{\boldsymbol{\theta}}$, is discretized into N waypoints. EESTO then seeks to iteratively optimize the equation

$$\min_{\tilde{\boldsymbol{\theta}}} \mathbb{E}\left[\sum_{i=1}^{N} q(\tilde{\boldsymbol{\theta}}_i) + \frac{1}{2}\tilde{\boldsymbol{\theta}}^\intercal \mathbf{R}\tilde{\boldsymbol{\theta}}\right], \tag{1}$$

where $\tilde{\boldsymbol{\theta}}$ represents a noisy trajectory with mean $\boldsymbol{\theta}$ and variance Σ, $q(\tilde{\boldsymbol{\theta}}_i)$ is a state dependent cost function and $\mathbf{R}$ is a matrix where $\mathbf{R} = \mathbf{A}^\intercal\mathbf{A}$ and $\mathbf{A}$ is a constant finite differencing matrix such that $\ddot{\boldsymbol{\theta}} = \mathbf{A}\boldsymbol{\theta}$. This selection is made so that $\mathbf{R}$ approximates the control costs. To remove the assumption that the waypoints are equally spaced in time we augment $\mathbf{A}$ so that $\mathbf{A} = \mathbf{TD}$ and,

$$\mathbf{D} = \begin{bmatrix} -2 & 1 & 0 & \\ 1 & -2 & 1 & \cdots \\ 0 & 1 & -2 & \\ & \vdots & & \ddots \end{bmatrix} \quad \mathbf{T} = \begin{bmatrix} 1/t_1^2 & 0 & \cdots \\ 0 & 1/t_2^2 & \\ \vdots & & \ddots \end{bmatrix}, \tag{2}$$

where $\mathbf{D}$ is a constant center finite differencing matrix and $\mathbf{T}$ is the step size which changes each iteration. At each iteration, t_i is updated based upon the average travel times found in the previous iteration for the waypoints involved in row i of the $\mathbf{D}$ matrix. We extend the waypoint formulation used in STOMP to accommodate this extra information by including the travel time to the following waypoint as in Kruger et al. [3].

The optimization problem in Equation 1 is solved by approximating the gradient of the cost function using a weighted combination of explored noisy paths. These noisy paths are calculated by sampling perturbations from a zero mean normal distribution with covariance $\mathbf{R}^{-1}$ which are added to the current path. In previous work this covariance could be computed prior to execution, but here must be calculated each iteration. EESTO calculates this inverse in a computationally tractable way by factoring $\mathbf{R}^{-1}$ as:

$$\mathbf{R}^{-1} = (\mathbf{A}^\intercal\mathbf{A})^{-1} = \mathbf{D}^{-1}(\mathbf{T}^2)^{-1}(\mathbf{D}^\intercal)^{-1}. \tag{3}$$

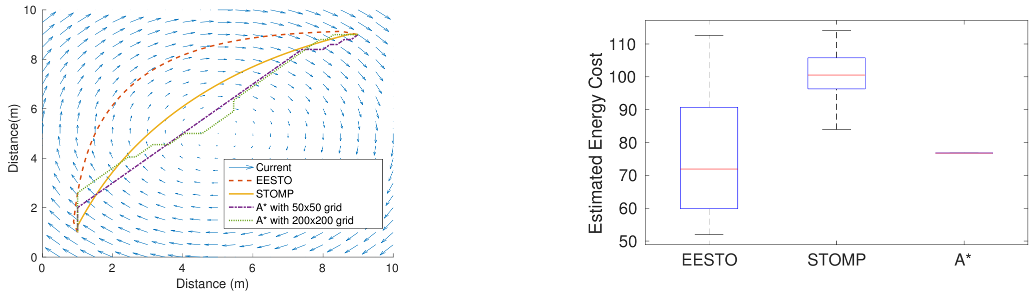
Both $\mathbf{D}^{-1}$ and $(\mathbf{D}^\intercal)^{-1}$ can be precomputed and $\mathbf{T}^2$ is a diagonal matrix whose inverse can be quickly calculated.

A. Cost Function

We now devise a suitable cost function for performing energy-efficient path planning using this general framework. Our cost function seeks to balance travel time and energy expenditure by attempting to travel at speeds close to or greater than the vehicle's maximum velocity, while minimizing the energy expended. The cost function is defined as:

$$q(\boldsymbol{\theta}_i) = C_s + C_e + C_o \tag{4}$$

$$C_s = \begin{cases} 0, & \text{if } V_r \leq V_{max} \text{ and } V_{abs} > V_{max} \\ \exp(V_{max} - V_r), & \text{if } V_r \leq V_{max} \\ l + (V_r - V_{max})l^2, & \text{if } V_r > V_{max} \end{cases} \tag{5}$$



(a) Comparison of Paths

(b) Path Energy

Fig. 1: Shown in (a) is the simulated environment. Shown in (b) is the energy used by the paths produced by the three planning methods. Paths produced by EESTO are not significantly more energy efficient than those given by A*, however these paths can be more reliably executed. EESTO takes 0.77 seconds on average in comparison to STOMP which takes 0.26s on average, and A* took 3.54s and 348.33s for grid sizes of 50x50 and 200x200 respectively.

$$C_e = \exp(E_{with} - E_{without}) \tag{6}$$

where C_s is the cost due to the speed, C_e is the cost due to the energy, and C_o is the obstacle cost represented by a large step if the waypoint is inside an obstacle. When calculating C_s, V_r is the required velocity for the motors to provide, V_{max} is the maximum velocity the motors can provide, V_{abs} is the absolute velocity that the vehicle is traveling, and l is some large number selected to introduce a step cost when the motors are required to provide a speed that they cannot achieve. C_e is calculated as the difference in energy cost when the given waypoint is present and when it is not. The energy cost is calculated in the same manner as in [6], where the drag force is assumed to dominate the inertial forces and as such the energy cost is:

$$E = c_d V_r^3 t, \tag{7}$$

where c_d can be equal to the actual drag coefficient or used to tune the cost function, and t is the travel time for the relevant section of the path.

III. Preliminary Results

A. Simulated Environments

The simulated ocean current environment can be seen in Figure 1 (a). The ocean current velocities range from $0\frac{m}{s}$ at (5,5) to $1\frac{m}{s}$ along the edges, representing half of the vehicle's maximum speed. Figure 1 (b) shows the results for 100 statistical runs for both EESTO and STOMP as well as the energy cost for the path produced by a simple A* search. While the approximated energy costs for A* and EESTO are quite close, the paths produced by EESTO, seen in Figure 1 (a), can more realistically be carried out on an actual vehicle because they do not require sharp changes in direction. Additionally, the mean run time for EESTO is 0.77 seconds, which allows for both multiple instances of the algorithm to be run and the lowest energy path selected when initially planning, as well as dynamic replanning as the environment is sensed more accurately.

B. Simulated Real World

EESTO was also used to plan a roughly 30 kilometer path off the coast of California using data from the ROMS ocean current data set for January 21, 2013. The path evolution and the final path found can be seen in Figure 2. The path in Figure

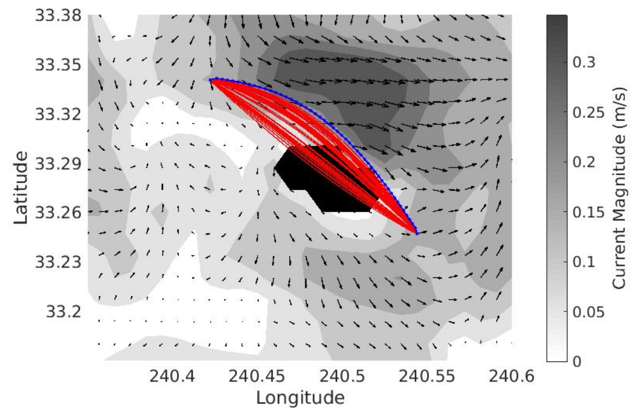


Fig. 2: Paths planned by EESTO using historical ocean current data. Red paths are the solutions found at the end of each iteration. The final path is represented in blue.

2 represents a standard path around the island produced by EESTO. The algorithm is able to both avoid the obstacle of the island, in an average of 35 iterations, and correctly identify that the vehicle can leverage the stronger currents north of the island to use less energy when traveling.

IV. Conclusion

We have presented a new algorithm for motion planning that can respect travel time variations. This algorithm is able to produce energy efficient paths while avoiding much of the necessary discretization introduced by A* like algorithms, with results from both simulated and real world scenarios.

Our current work is looking at how to use this algorithm more effectively in an online manner alongside execution. EESTO works by improving existing paths and so is ideally formulated for online path optimization and is computationally efficient enough that dynamic replanning of paths based upon environmental sensing is feasible. Additionally, this formulation could lead to a principled way of allowing for the trade off between exploration and exploitation when there is uncertainty in predictions.

Acknowledgments

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